Intellectual Property Rights and Trade Liberalization from the North-South Perspectives

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Abstract

This paper studies whether there are optimal levels of intellectual property rights (IPR) in large emerging market economies, both from an advanced economy and a large emerging market economy’s perspectives. I develop a North-South model of endogenous technological change with firm heterogeneity, in which innovation and imitation co-exist in the South. I show that it is the growth rather than the level of production costs that matters for determining the effect of strengthening Southern IPR on the South’s growth rate. The effect is positive if the growth of production costs is not excessively fast. Following this result, I show that there exists an inverted U-shaped relationship between consumer welfare and Southern IPR, arising from the trade-off between the variety expansion effect and the resource allocation effect due to increased R&D investment, and that the optimal Southern IPR from the North and the South’s perspectives can be different. Quantitative analysis using data from the US and China shows that by increasing Southern IPR to the optimal level, US and China’s welfare gains are 0.4% and 7%, respectively. Together with trade liberalization, the respective welfare gains can be up to 6% and 13%. Indeed, trade liberalization can potentially raise the optimal IPR, and can affect the distribution of welfare gains between countries. Also, due to firm heterogeneity, IPR and trade openness have offsetting effects on the South’s average firm productivity for production. These results call for the importance of having a policy mix of trade openness and IPR.

Keywords: Innovation, Imitation, Intellectual Property Rights, Endogenous Growth, Transitional Dynamics, International Trade, Heterogeneous Firms.

JEL classification: F12, F13, F43, O31, O33, O34, O41

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1 Introduction

Is there an optimal level of intellectual property rights for a large emerging market economy? Does a technological advanced country always like to see a strengthening in intellectual property rights in the large emerging economy? Given the sheer size of large emerging market economies such as China and India (henceforth the South), the optimal level of intellectual property rights (IPR) in these economies is of global interest. Stricter enforcement of IPR in the South not only has growth and welfare implications on its own economy but also on the advanced economies (henceforth the North). Moreover, trade policies may have a role to play in affecting the optimal level of IPR.

This paper studies optimal IPR and trade openness in the North-South context using a model of endogenous technological change with firm heterogeneity, in which innovation and imitation co-exist in the South. From both the North and the South’s perspectives, this paper provides a theoretical and quantitative assessment of strengthening intellectual property rights and trade liberalization along the transition and balanced growth paths, in which the welfare effects account for the dynamic effects arising from endogenous changes in the growth rates.

Studies in the trade and IPR literature mostly focus on the welfare effects that are derived from the balanced growth path dynamics (e.g. Lai (1998), Eaton and Kortum (1999), Glass and Saggi (2002), Qiu and Lai (2004), Grossman and Lai (2004), Glass and Wu (2007), and Lai and Yan (2013)). The trade literature also focuses mostly on static gains from trade (e.g. Arkolakis, Costinot, and Rodriguez-Claire (2012) and Hsieh and Ossa (2016)). Contributing to the literature, the results in this paper suggest that ignoring the transitional dynamics can lead to biased estimates of welfare effects. Although some of the studies in the growth literature also explore transitional dynamics (e.g. Grossman and Helpman (1991) and Helpman (1993), Connolly and Valderrama (2005), Mondal and Gupta (2009), and Dinopoulos and Segerstrom (2010)), they usually assume the South as an imitator country without innovation, which undermine the possibility of leapfrogging by a large emerging market economy.

This paper builds on the endogenous technological change literature (e.g. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Acemoglu et al. (2013), Perla and Tonetti (2014), and Wu (2015)), and the modeling of firm heterogeneity and trade follows Melitz (2003). Economic growth in the model is generated by the expansion in the variety of intermediate goods, and the decisions on research and development (R&D) are affected by IPR and trade openness. The co-existence of innovation and imitation in the South captures the possibility that a laggard

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1 This is in contrast to many of the studies in the literature which study IPR issues using semi-endogenous growth models. In this class of models, which includes Jones (1995b), Dinopoulos and Segerstrom (2010), and Gustafsson and Segerstrom (2010 and 2011), economic growth hinges on growth in the R&D sector, which grows alongside R&D employment in equilibrium. Although, in these models, economic policies can generate long-run welfare effects through changes in the level of consumption, they generally do not have any long-run growth effects, unless the policies can change the labor force growth. On the other hand, Ha and Howitt (2007) show that fully-endogenous growth models are better than the semi-endogenous growth models at forecasting long-run swings in growth rates.
country can leapfrog.

Firm heterogeneity is especially important when the possibilities of innovation and imitation co-exist. As discussed in Melitz and Redding (2015), due to the endogenous firm selection, the welfare gains from reducing trade costs are larger in heterogeneous firm model than in homogeneous firm model. In this paper, IPR can affect average productivity through the firm distribution, and the existence of imitation can change the competition faced by incumbents over time, which adds further dynamics to the evolution of firm distribution. Thus, having firm heterogeneity in the model improves the measure of aggregate welfare and provides new policy implications.

Based on the North-South model, I derived several theoretical results. I show that it is the growth rather than the level of production costs that matters for determining the effect of strengthening Southern IPR on the South’s growth rate. The effect is positive if the growth of production costs is not excessively fast. Following this result, I show that these exists an inverted U-shaped relationship between consumer welfare and Southern IPR for both the North and the South, which arises from the trade-off between a variety expansion effect and a resource allocation effect from consumption to R&D investment as IPR strengthens. The variety expansion effect refers to the increase in the R&D growth rate and effective consumption due to faster output growth when IPR is strengthened. The resource allocation effect refers to the increase in R&D expenditure which reduces resources available for consumption. Due to the two offsetting effects, there exists optimal Southern IPR that maximizes consumer welfare, but the optimal Southern IPR from the North and the South’s perspectives can be different given the different economic conditions. Meanwhile, trade liberalization is always welfare improving in the model.

In the quantitative analysis, the US data is used for representing the North and the Chinese data for representing the South. China is a rising large emerging market economy which suits well the model description, particularly given its size and importance in the global supply chain in the past two decades. I treat US as the North due to its size and the level of technological development, and it is also the final export destination or the source of final demand for many of the Chinese exports. Based on a calibrated version of the model, I compute the equilibrium transition and balanced growth paths under the benchmark case and counterfactual scenarios from a scheme of IPR and iceberg trade costs. I quantify the welfare gains and show that there exist optimal levels of IPR from the North and the South’s perspectives due to the inverted U-shaped relationship between consumer welfare and Southern IPR.

I go further and compute welfare gains from strengthening IPR together with trade liberalization. I show that trade openness can potentially raise the optimal IPR, depending on the relative importance of trade liberalization and IPR protection for a country. By increasing Southern IPR to the optimal level, China’s welfare gain is 7%, and can be up to 13% together with trade liberalization. The respective welfare gains for the US are 0.4% and 6%. The trade openness and

\[ \text{He, Liao, and Wu (2015) discuss the features of Chinese exports.} \]
IPR policy mix yields a US welfare gain that is 15 times of that when there is a strengthening in Southern IPR alone, whereas China’s has doubled. This means that trade liberalization also affects the distribution of welfare gains between countries. Moreover, due to firm heterogeneity, IPR and trade openness have offsetting effects on the South’s average firm productivity for production. As such, policymakers who concern global IPR policies should consider trade policies together to come up with a welfare-improving policy mix. This policy implication answers the question as to whether international coordination of IPR such as TRIPS needs to occur at the WTO (see Saggi (2016)).

The next section presents a North-South model of endogenous technological change with firm heterogeneity. Section 3 define the equilibrium and characterizes the balanced growth path of the model. Section 4 shows the theoretical results on the welfare effects of strengthening IPR and trade liberalization. Section 5 quantifies the welfare effects using data on the US and China. Section 6 concludes.

2 The Model

2.1 The Environment

There are two countries in the world economy, the North and the South, which are at different stages of economic development. One can think of the North representing the advanced economies, and the South representing a large emerging market economy. Let country $i \in \{n, s\}$, where $n$ and $s$ denote the North and the South, respectively. The South is different from the North in two aspects: a lower unit cost of production and weaker enforcement of IPR. There is no aggregate uncertainty. Time is discrete and is indexed by $t \geq 0$. Population sizes $\bar{L}_i$ are fixed without loss of generality.\footnote{The impacts of changes in demographics on transition and balanced growth paths are beyond the scope of this paper.}

A representative household in country $i$ at time $t$ derives utility from the consumption of a final good, $C_{it}$. It’s preference is given by the expected discounted utility:

$$U_i = \sum_{t=0}^{\infty} \beta^t u(C_{it}),$$

where

$$u(C_{it}) = \frac{C_{it}^{1-\theta} - 1}{1 - \theta}.$$  

The household supplies one unit of labor services per unit of time, so that the size of the labor force $L_i$ is the same as the size of population $\bar{L}_i$. Household’s budget constraint is given by:

$$B_{it+1} = w_{it}L_i + (1 + \mu_{it})B_{it} + Z_{it} - C_{it},$$

where $w_{it}$ denotes the real wage, $\mu_{it}$ denotes the interest rate, $B_{it}$ denotes the assets owned by the households, and $Z_{it}$ denotes transfers from the intermediate goods sector. The representative
household earns labor income, investment income, and collects transfers from the intermediate goods sector. There is no capital flow across the border for investment purpose, so the interest rates can be different in the two countries.

There are two production sectors in each economy: a perfectly competitive final good sector and an intermediate goods sector with differentiated goods. All the firms in these two sectors are owned by the households in their respective countries. The final good $Y_{it}$ is the numeraire good and is sold at unit price. Final good are identical across economies and can be traded without cost.

The final good for country $i$ is produced using labor and a continuum of intermediate inputs $Q_{li}$, where $l \in [0, N_{it}]$. $N_{it}$ is the variety of intermediate goods that can be used as inputs in country $i$ at time $t$, which includes intermediate goods from domestic and foreign firms. As seen below, not all intermediate goods variety is used as inputs in the final good production, as some of the intermediate goods are created but are not produced. The aggregate production function for the final good for country $i$ is given by:

$$Y_{it} = A_i L_i^{1-\alpha} \int_0^{N_{it}} Q_{it}^{\alpha} dl,$$

(3)

where $A_i$ is the aggregate productivity parameter, and $\alpha \in (0, 1)$ controls the elasticity of substitution between intermediate goods, $\epsilon = 1/(1 - \alpha) > 1$.

Final output can be used for aggregate consumption, $C_{it}$, the spending on intermediate goods, $Q_{it}$, which includes all $Q_{li}$, investment in research and development (R&D), $R_{it}$, and net exports, $E_{it}$:

$$C_{it} + R_{it} + E_{it} \leq Y_{it} - Q_{it}.$$

(4)

Net exports ($E_{it}$) are equal to the negative of net profits repatriated from abroad. Repatriated net profits here refer to the net profits earn by exporting intermediate firms, which are paid in the form of the final good.

Firms in the intermediate goods sector invest to obtain blueprints, either through innovation or imitation, and are ready to produce and sell differentiated products. They are single-product firms, and are heterogeneous in their cost of production. Northern and Southern intermediate firms are subject to different degrees of IPR protection. All intermediate firms originating from the North are innovators as IPR is strictly enforced among these firms. The IPR protection in the South is weaker. This allows for the presence of imitation.

Each intermediate firm faces a two-stage problem — the R&D stage (first stage) and the production stage (second stage). In the first stage, each Northern firm must first invest in innovation and to obtain a perpetual patent on a blueprint by paying a one-time cost, $\eta_{nt+1}$. The entry of Northern innovators depends on expected profits and $\eta_{nt+1}$. Southern firms must also invest upfront, in which a blueprint can be obtained either by innovation or imitation. The entry of Southern innovators depends on expected profits and a one-time cost of innovation, $\eta_{nt+1}^I$. Potential imitators
also need to pay a one-time entry cost, $\eta_{st+1}^{imit}$, to enter. The entry of imitators is governed by the probability of imitation, $\phi_{t+1}$, which is a function of the state of the economy. $\eta_{nt+1}$, $\eta_{st+1}^{imit}$, and $\eta_{st+1}$ are also functions of the state of the economy at $t$.

Once a Northern product has been imitated by a Southern firm, no other Southern firms will copy that same product unless all Northern products have been imitated. Instead of competing with other imitators over the same product and driving prices down to the marginal costs, a new imitator can imitate an uncopied product instead and can potentially enjoy non-negative profits over the newly-imitated product. Southern imitators also face an exogenous probability of exit, $b_s$, which represents the degree of IPR enforcement in the South. This is the IPR policy tool in this model. In the case of successful IPR enforcement, a Southern imitator would cease to operate in the production stage forever but the imitated technology remains in the South’s pool of technology, and this event happens with probability $b_s$.

In the second stage, each intermediate firm, whether it is an innovator or an imitator, learns $a$, a firm-specific unit cost component which is denominated in terms of the final good, and is heterogeneous across firms. It is drawn from a Pareto distribution:  

$$G_i(a) = Pr(a < a) = \left(\frac{a}{a_0}\right)^k, \quad a \in [0, a_0],$$

where $a_0$ and $k$ are the scale and shape parameters of the distribution, respectively. An intermediate firm only draws the unit cost parameter once in its lifetime. Northern and Southern firms choose either to serve the domestic market only, to serve both the domestic and foreign markets, or not to produce at all, depending on their unit cost draw.

The production function of an intermediate good is linear. The unit cost of production for a Northern firm is $a$, whereas the unit cost of production for a Southern firm is $\gamma_t a$. $\gamma_t$ reflects the difference in the unit cost of producing the same product in the North and the South in order to reflect the cost advantage in the South. I assume that $\gamma_t$ is an increasing function of the level of development in the South relative to the North, so that the cost advantage will diminish as the South is catching up to the North. A firm that serves the domestic market produces its product using domestic final good. To serve the foreign market, exporters first purchase final good inputs from the foreign country, and then produce the intermediate goods at home and ship the products to the foreign country by incurring an iceberg trade cost, $\tau$. This assumption simplifies the market clearing of world markets, and avoids the resource constraint from not having enough final inputs at

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4Usually, the Pareto distribution is given by $Pr(X > x) = (x/x_0)^{-k}$. If we substitute $x = 1/a$ and $x_0 = 1/a_0$ the distribution becomes $Pr(a < a) = (a_0/a)^{-k} = (a/a_0)^k$. Also, $k$ needs to be greater than 2 for the mean and variance of the distribution to be finite.

5Most emerging market economies have seen their costs of production increasing along with their phase of development. Even if their productivity continues to rise and technology has been catching up to the frontier, other factors such as labor law reforms or the implementation of pollution prevention law can raise the production costs, as measured for instance by unit labor cost.
home to produce goods for exports. The iceberg trade cost consists of tariffs, costs due to non-tariff barriers, as well as technological costs such as transportation and inventory-carrying costs.

Innovators who produce goods that have not been imitated engage in monopolistic competition. Innovators whose goods have been copied may engage in price competition with the imitators, or exit, depending on $\gamma_t$. For example, even if the product of a Northern firm has been imitated, the firm does not necessarily need to exit the market as long as it can deter entry and earn non-negative profits. This can happen if $\gamma_t$ is sufficiently large, meaning that the cost advantage in the South has diminished. Prices need not be pushed down to the marginal costs. For example, a Southern imitator can charge a price just below the break-even price of a Northern innovator in order to deter entry, while still earning a positive profit. If $\gamma_t$ is very low, a Southern imitator may charge a monopoly price while still deterring the entry of Northern innovator.

A per-period fixed cost $F_{di}$ is paid by the domestic intermediate firms from country $i$ to serve the domestic market, where subscript $d$ denotes “domestic”. To serve abroad, exporters originating from country $i$ pay an additional per-period fixed cost $F_{xi}$, where subscript $x$ denotes “export”. Generally speaking, the fixed costs consist of costs to set up and maintain production facilities, distribution costs, lobbying costs, and other policy-related costs. To guarantee non-negative average net profits for exporters, the per-period fixed costs satisfy the following condition:

$$\frac{\Lambda_j}{\Lambda_i} F_{di} < \tau^{\epsilon-1} F_{xi},$$

where $\Lambda_i = A_i^{1-\alpha} \alpha^{\frac{2}{3}} L_i$ and similarly for $\Lambda_j$.

The number of intermediate firms originating from the North and the South will be expanding over time at rate $g_{it}$. $M_{it}$ denotes the number of firms originating from country $i$. From the production standpoint, $M_{it}$ includes domestic firms, $N_{dit}$, and exporters, $N_{xit}$, that produce positive output, and the idle firms which have obtained a blueprint but are not productive enough to make non-negative profit, which is $M_{it} - N_{dit} - N_{xit}$. From the R&D standpoint, $M_{nt}$ includes all innovators originating from $n$, and $M_{st}$ includes all innovators, $M_{st}^I$, and imitators, $M_{st}^{imit}$, that are originating from $s$, regardless of whether these intermediate firms are profitable.

Relating the notations to the aggregate final good production (3), the variety of intermediate goods ($N_{it}$) used for production is equal to the sum of the domestic variety ($N_{dit}$) and the imported goods from foreign exporters ($N_{xjt}$) from country $j \neq i$ that produce and earn non-negative profits in the second stage. The total per-period fixed costs paid by domestic firms and foreign exporters are $N_{dit} F_{di}$ and $N_{xjt} F_{xj}$, respectively. These fixed costs are part of domestic households’ income, and are collected in the form of transfers, $Z_{it}$, for households in country $i$, as appeared in the budget constraint (2). I present each sector’s problem in the sections below.
2.2 Final Good Sector

Final good producers minimize costs by choosing labor and intermediate inputs, subject to production function (3), taking prices of intermediate goods as given. Specifically,

$$\min_{L_i, X_{li}} Y_{it} - w_{it} L_i - \int_0^{N_{it}} p_l Q_{it} dl.$$  (7)

The producers’ problem yields the equilibrium real wage and the demands for intermediate goods. The real wage is given by:

$$w_{it} = (1 - \alpha) \frac{Y_{it}}{L_i}. $$  (8)

The demand for intermediate good $l$ by final good producers in country $i$ is given by:

$$Q_{li}(p_l) = \left( \frac{A_i \alpha}{p_l} \right)^{\frac{1}{1-\alpha}} L_i,$$  (9)

where the price elasticity of demand for each intermediate good $l$ is $-1/(1 - \alpha)$.

2.3 Intermediate Goods Sector — Stage 2: Production

Given that each intermediate firm faces a two-stage problem, the problem is solved by backward induction. In the second stage, firms have obtained a blueprint and are ready for production. Each firm draws its unit cost from the Pareto distribution (5). Since all intermediate goods enter the final good aggregate production function (3) the same way, with the only difference being the unit cost draw $a$ for each intermediate good, henceforth I index intermediate goods with their unit cost $a$ instead of good $l$.

To decide whether to produce and sell in the domestic market at each period $t$, firms compare their potential operating profits with the per-period fixed costs. Firms can also choose to export if they can make non-negative profits by exporting. A firm from country $i$ with unit cost $a$ maximizes its per-period profit by choosing an optimal price, $p_{hit}$ for $h \in \{d, x\}$. A domestic/exporting firm’s problem is given by:

$$\max_{p_{hit}} \left( p_{hit} - \tau^q \gamma_t^q a \right) Q_h(p_{hit}) - F_{hi}$$  (10)

subject to the demand function (9) from the final good producers’ problem. I denote $q \in \{0, 1\}$, where $q = 0$ for a domestic firm’s problem so that $\tau = 1$, and $q = 1$ for an exporter’s problem which incurs an iceberg trade cost ($\tau > 1$). Also, $\gamma_t^d = 1$ for Northern firms and $\gamma_t^s = \gamma_t$ for Southern firms. $\gamma$ is an increasing function of $M_{st}/M_{nt}$, which reflects the rising cost of production as the level of development in the South is catching up to the North. $^6$ Specifically, it is given by:

$$\gamma_t = \lambda \left( \frac{M_{st}}{M_{nt}} \right)^{\sigma},$$  (11)

where $\lambda > 0$ and $\sigma > 0$.

$^6$See footnote (5) for explanation on this assumption.
The optimal prices, conditional on being imitated or not, are given in Table 1. Northern firms charge monopoly prices when their products have not been copied, and Southern innovators always charge monopoly prices. These prices are shown in the top panel in Table 1. If the Northern products have been copied by Southern imitators, different prices can be charged, depending on the conditions listed in the first column in the bottom panel of Table 1. Under the first condition, a Southern imitator charges a monopoly price on its exports (or the North’s imports), given that the monopoly price is below North’s unit cost of production, \( \alpha \), for product \( a \). Under condition (ii), because the monopoly price will be too high, a Southern imitator charges a competitive price that is slightly below North’s break-even price, in which I put it as the North’s break-even price, \( \alpha \), for simplicity. Under the third and fourth conditions, the unit cost of a Southern exporting imitator is too high, so the Northern innovator of product \( a \) can deter the Southern imitator’s entry by charging a price right below the Southern imitator’s unit cost under condition (iii), and a monopoly price under condition (iv). Similarly, a Southern imitator can deter the Northern
innovator of product \( a \) to enter the South market given that its unit cost is low enough, and charges a monopoly price and a competitive price below the Northern innovator’s unit cost under conditions (v) and (vi), respectively. When the North’s unit cost is low enough to deter a Southern imitator’s entry, Northern innovators charge a competitive price right below South’s unit cost under condition (vii), and a monopoly price under condition (viii).

Under all the conditions, there is a marginal firm that is indifferent between entering or not, because it earns zero profit from production and sales. Firms that have a unit cost draw below the marginal firm’s make positive profits. The marginal firms with unit cost draw that are right at the cut-off unit costs can be found by the zero-profit conditions:

\[
\pi_I^I(a_I^{hit}) = F_{hit} \tag{12}
\]

for Northern and Southern innovators, and

\[
\pi_{hst}^{imit}(a_{hst}^{imit}) = F_{hst} \tag{13}
\]

for Southern imitators, \( h \in \{d, x\} \), in which \( a_I^{hit} \) and \( a_{hst}^{imit} \) denote the cut-off unit costs of the marginal firms. Also,

\[
\pi_I^{hit}(a_I^{hit}) = (p_I^{hit} - \tau q_j a_I^{hit})Q_j(p_I^{hit}) \tag{14}
\]

\[
\pi_{hst}^{imit}(a_{hst}^{imit}) = (p_{hst}^{imit} - \tau q_j a_{hst}^{imit})Q_j(p_{hst}^{imit}) \tag{15}
\]

denote the operating profits of the marginal firms at the optimal prices. The optimal prices are denote by “ \( p^* \) ”, and \( q \in \{0, 1\} \) for domestic and exporting firms, respectively. The cut-off unit costs can be different across conditions (i) to (viii) listed in Table 1. Firms that have unit cost draws above the cut-off unit costs will not produce and sell in the respective markets, and the technology will remain idle.

Condition (6) on the per-period fixed costs, together with (12) and (13), imply that:

\[
a_{xit}^I < a_{dit}^I, \tag{16}
\]

\[
a_{xist}^{imit} < a_{dist}^{imit}. \tag{17}
\]

A higher fixed cost to export requires more productive firms to enter, which is the same as in Melitz (2003). I further expand the cut-off unit costs to distinguish cut-offs under monopoly and competitive pricing:

\[
a_I^{I,c} < a_I^{I,m}, \tag{18}
\]

\[
a_{hst}^{imit,c} < a_{hst}^{imit,m}, \tag{19}
\]

where \( a_{hit}^{m} \) denotes cut-off unit cost under monopoly pricing and \( a_{hit}^{c} \) denotes cut-off unit cost under competitive pricing. These two conditions mean that when the competitive prices are optimal,
the cut-off unit costs are smaller than when the monopoly prices are optimal. This is because competitive profits are below monopoly profits, which means that only the more productive firms will enter and earn positive profits.

2.3.1 Fixed Costs and Scale Effect Corrections

The fixed costs are defined in such a way as to eliminate scale effects in the output growth rate. Following Wu (2015), I assume that fixed costs are given by $F_{hi} = \frac{f_{hi}}{(K_{it}/Q_{it})}$ for $h \in \{d, x\}$ and $i \in \{n, s\}$. $K_{it}$ is an index of a country’s stock of knowledge. $K_{it}/Q_{it}$ is the stock of knowledge per intermediate good originating from country $i$ that is used in country $i$’s final good production. This modification implies that the entry cost is proportional to the relative stock of knowledge — the larger the relative stock of knowledge the lower the entry cost. I assume $K_{it} = \bar{a}_{it}^{1-\epsilon} M_{it}$, where $\bar{a}_{it}^{1-\epsilon}$ is an index of average productivity of intermediate firms in country $i$ used for producing the final good in $i$. Also, recall that $M_{it}$ is the number of intermediate firms originating from $i$, including all firms that produce and sell or remaining idle. Such a modification is consistent with Barro and Sala-i-Martin (2004) and Dinopoulos and Unel (2011). From Section 2.5, $Q_{it} = \alpha Y_{it}$, where $Y_{it}$ is given by (26) in below. Per-period fixed costs turn out to be constant, given by:

$$F_{hi} = \frac{f_{hi}}{(1-\epsilon)\alpha^{2\epsilon-1} L_i}. \tag{20}$$

2.3.2 Aggregation

Intermediate goods are used as inputs in the production of final good. Hence, the Northern aggregate production function for the final good (3) can be rewritten as:

$$Y_{it} = A_i L_i^{1-\alpha} \left[ \int_0^{\alpha_{xjt}} Q_i(a) \frac{g(a)}{G(a_{xjt})} N_{xjt} \, da + \int_0^{\alpha_{cjt}} Q_i(a) \frac{g(a)}{G(a_{cjt})} N_{cjt} \, da \right]
$$

$$+ \int_0^{a_{cjt}} Q_i(a) \frac{g(a)}{G(a_{xjt})} N_{fjt} \, da + \int_0^{a_{fjt}} Q_i(a) \frac{g(a)}{G(a_{fjt})} N_{fjt} \, da
$$

$$+ \int_0^{a_{imit,m}} Q_i(a) \frac{g(a)}{G(a_{imit,m})} N_{imit,m} \, da + \int_0^{a_{imit,c}} Q_i(a) \frac{g(a)}{G(a_{imit,c})} N_{imit,c} \, da
$$

$$+ \int_0^{a_{imit,m}} Q_i(a) \frac{g(a)}{G(a_{imit,m})} N_{imit,m} \, da + \int_0^{a_{imit,c}} Q_i(a) \frac{g(a)}{G(a_{imit,c})} N_{imit,c} \, da \right], \tag{21}

The scale effect is referred to the feature in endogenous growth models in which long-run growth rate increases with population size or resources devoted to R&D, such as in Romer (1990) and Grossman and Helpman (1991). This feature, however, is not supported by the data, as shown in Jones (1995a). Even though semi-endogenous growth models such as in Jones (1995b) and Segerstrom (1998) have corrected for the scale effect, these models do not have long-run growth effects, given that the balanced growth rate is determined by the exogenous employment growth in the R&D sector. In turn, the result of constant R&D employment growth is not supported by Ha and Howitt (2007), and support fully-endogenous growth models. A similar discussion can be found in Wu (2015).
where $G(\cdot)$ is a Pareto distribution given by (5), $g(a)$ is the a density function of the Pareto distribution. $g(a)/G(\cdot)$ represents the density functions conditional on firms survival in the production stage. The final output in $i$ is produced using labor ($L_i$) and intermediate inputs from the domestic innovators (first line), imports from innovators originating from country $j$ (second line), domestic imitators (third line), and imports from imitators originating from country $j$ (last line). Note that some of the integrals in (21) are equal to 0, depending on the existence of domestic imitators and entry deterrence following the optimal prices given in Table 1. Also, some of the integrals in (21) are equal to 0, depending on the existence of domestic imitators and entry deterrence following the optimal prices given in Table 1. Also, 

\[
N_{dit}^{I,m} = (1 - \phi_{it})G(a_{dit}^{I,m})M_{it}^{I}, \quad N_{dit}^{I,c} = \phi_{it}G(a_{dit}^{I,c})M_{it}^{I}, \quad (22)
\]

\[
N_{xjt}^{I,m} = (1 - \phi_{jt})G(a_{xjt}^{I,m})M_{jt}^{I}, \quad N_{xjt}^{I,c} = \phi_{jt}G(a_{xjt}^{I,c})M_{jt}^{I}, \quad (23)
\]

\[
N_{dit}^{imit,m} = (1 - b_s)G(a_{dit}^{imit,m})M_{it}^{imit}, \quad N_{dit}^{imit,c} = (1 - b_s)G(a_{dit}^{imit,c})M_{it}^{imit}, \quad (24)
\]

\[
N_{xjt}^{imit,m} = (1 - b_s)G(a_{xjt}^{imit,m})M_{jt}^{imit}, \quad N_{xjt}^{imit,c} = (1 - b_s)G(a_{xjt}^{imit,c})M_{jt}^{imit}, \quad (25)
\]

where the probability of imitation $\phi_{it} = \phi_t$ due to Northern innovators facing Southern imitation, and $\phi_{st} = 0$ given that there is no Northern imitation faced by Southern innovators. Moreover, $M_{it}^{I} = M_{it}$ and $M_{it}^{imit} = 0$ as all Northern firms are innovators.

Given equations (22) to (25), the number of domestic firms originating from $i$ is $N_{dit} = N_{dit}^{I,m} + N_{dit}^{I,c} + N_{dit}^{imit,m} + N_{dit}^{imit,c}$, and the number of foreign exporters from $j$ that enter country $i$’s aggregate production is $N_{xjt} = N_{xjt}^{I,m} + N_{xjt}^{I,c} + N_{xjt}^{imit,m} + N_{xjt}^{imit,c}$. Also, $N_{it} = N_{dit} + N_{xjt}$. Equation (21) can be simplified by substituting $Q_t(a)$ from equation (9), the optimal prices in Table 1, and using $g(a)$ derived from the Pareto distribution (5), and (22) to (25). This becomes:

\[
Y_t = A_t^{1-\alpha} \alpha^{\frac{2\alpha}{1+\alpha}} L_t N_t \tilde{a}_t^{1-\epsilon}. \quad (26)
\]

where $\epsilon = 1/(1-\alpha)$ is the elasticity of substitution between intermediate goods. The average unit costs of intermediate goods used in country $i$’s final good production, $\tilde{a}_t$, is given by:

\[
\tilde{a}_t = \left[ \frac{N_{dit}^{I,m}}{N_{it}} (\gamma_t \tilde{a}_{dit}^{I,m})^{1-\epsilon} + \frac{N_{dit}^{I,c}}{N_{it}} (\gamma_t \tilde{a}_{dit}^{I,c})^{1-\epsilon} + \frac{N_{xjt}^{I,m}}{N_{it}} (\tau \gamma_t \tilde{a}_{xjt}^{I,m})^{1-\epsilon} + \frac{N_{xjt}^{I,c}}{N_{it}} (\gamma_t \tilde{a}_{xjt}^{I,c})^{1-\epsilon} + \frac{N_{dit}^{imit,m}}{N_{it}} (\gamma_t \tilde{a}_{dit}^{imit,m})^{1-\epsilon} + \frac{N_{dit}^{imit,c}}{N_{it}} (\gamma_t \tilde{a}_{dit}^{imit,c})^{1-\epsilon} + \frac{N_{xjt}^{imit,m}}{N_{it}} (\tau \gamma_t \tilde{a}_{xjt}^{imit,m})^{1-\epsilon} + \frac{N_{xjt}^{imit,c}}{N_{it}} (\tau \gamma_t \tilde{a}_{xjt}^{imit,c})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \quad (27)
\]

where $\gamma_t^n = 1$ and $\gamma_t^s = \gamma_t$.

### 2.4 Intermediate Goods Sector — Stage 1: Innovation and Imitation

In the first stage, potential entrants in the intermediate goods sector decide whether to invest in R&D. I define the state $X_t \in \{M_{it}^{I}/M_{it}, M_{it}/M_{it}\}$. The R&D decisions affect $X_{t+1}$. Entrants in
the North are all innovators as IPR is strictly enforced on Northern firms. They devote resources to innovation by paying a one-time cost \( \eta_{nt+1} \) which is the same for all Northern entrants. It is given by:

\[
\eta_{nt+1} = f_n A_n^{1/\alpha} \alpha^{\alpha/2} \eta_n \cdot \left( \frac{M_{st+1}}{M_{nt+1}} \right)^{\psi_n},
\]

(28)

where \( f_n \) is constant. The cost of innovation is defined in such a way to eliminate scale effects in the output growth rate, consistent with the per-period fixed cost (20). Also, I assume \(-1 \leq \psi_n \leq \bar{\psi}_n\), with \( \bar{\psi}_n \leq 1 \). This means that \( \eta_{nt+1} \) can be an increasing and concave, or a decreasing and convex function of \( M_{st+1}/M_{nt+1} \). This captures the net effect of rising costs due to the increasing level of difficulty in R&D in the North from maintaining the North’s technological frontier ahead of the South, and of the decreasing costs due to technology spillover coming from the frontier technology in the North.\(^8\) The importance of having the upper bound \( \bar{\psi}_n \) will be explained below.

Similarly, Southern innovators pay a one-time cost of innovation which is given by:

\[
\eta_{st+1} = f_s A_s^{1/\alpha} \alpha^{\alpha/2} \eta_s \cdot \left( \frac{M_{st+1}}{M_{st+1}} \right)^{\psi_s},
\]

(29)

where \( f_s \) is constant. I assume \( 0 \leq \psi_s \leq \bar{\psi}_s \), with \( \bar{\psi}_s \leq 1 \), so that the cost is increasing and concave in \( M_{st+1}/M_{st+1} \). \( \psi_s \geq 0 \) is important for finding a condition in which growth would increase with strengthening IPR, which will be discussed in Section 4 below. Also, the reason for having \( \eta_{st+1}^s \) as a function of \( M_{st+1}/M_{st+1} \) instead of \( M_{nt+1}/M_{nt+1} \) as in (28) is to define the costs of R&D in the two economies so that they span the two state variables in the model independently. This assumption helps to simplify the method requires to solve the households’ problems in Section 2.5.

Southern imitators also pay a one-time cost \( \eta_{st+1}^i \) to imitate an uncopied Northern product. It is given by:

\[
\eta_{st+1}^i = f_s^i A_s^{1/\alpha} \alpha^{\alpha/2} \eta_s \cdot \left( \frac{M_{st+1}^i}{M_{st+1}} \right)^{\psi_s^i},
\]

(30)

where \( b_s \) is the exogenous probability of exit for Southern imitators, representing the strength of IPR in the South, and \( 1 - b_s \) is the probability of not being caught by IPR enforcement. Similarly, \( 0 \leq \psi_s^i \leq \bar{\psi}_s^i \), with \( \bar{\psi}_s^i \leq 1 \) so that the cost is increasing and concave in \( M_{st+1}^i/M_{st+1} \). The importance of \( \psi_s^i \geq 0 \) will be discussed in Section 4 below.

The ex-ante, expected net profit \( \tilde{\pi}_{st+1} \) for \( i \in \{n, s\} \) is equal to the sum of the innovators and imitators’ average net profits in the second stage problem, multiplied by the respective proportions

\(^8\)As discussed in Jones (1995b), both the possibilities of increasing and decreasing returns to R&D can be reasonable, though the author uses decreasing returns to R&D in the model. More recent study by Bollard, Klenow, and Li (2016) supports rising entry cost with the level of development as entrants are required to set up more technologically sophisticated operations.
of firms surviving in the second stage, which are also the probabilities of survival. It is given by:

\[
\tilde{\pi}_{it+1} = \frac{N_{i}^{I,m}}{M_{it+1}} \cdot \left[ \tilde{\pi}_{dit+1} \cdot \frac{N_{i}^{I,c}}{N_{i}^{I,m}} + \tilde{\pi}_{mit+1} \cdot \frac{N_{i}^{I,m} \cdot \tilde{\pi}_{mit+1}}{N_{i}^{I,m} \cdot \tilde{\pi}_{mit+1} + N_{i}^{I,c} \cdot \tilde{\pi}_{mit+1}} \right] + \frac{N_{imit,m}}{M_{it+1}} \cdot \left[ \tilde{\pi}_{mit+1} \cdot \frac{N_{imit,c}}{N_{imit,m}} + \tilde{\pi}_{imit,c} \cdot \frac{N_{imit,m} \cdot \tilde{\pi}_{imit,c}}{N_{imit,m} \cdot \tilde{\pi}_{imit,c} + N_{imit,c} \cdot \tilde{\pi}_{imit,c}} \right],
\]

for \( j \neq i \). \( \tilde{\pi}_{hit+1}^{I,i} \) denotes the average net profits earned in \( h \in \{d, x\} \) by the average innovator originating from \( i \) who charges either a monopoly price \( (m) \) or competitive price \( (c) \). Similarly, \( \tilde{\pi}_{hit+1}^{imit,i} \) denotes the average profits earned in \( h \) by the average imitator from \( i \) who charges either \( m \) or \( c \). Note that due to the probability of exit for Southern imitators, and that they may be charging competitive prices to deter Northern firms entry and earn non-monopoly profits, the expected profits for a Southern imitator must be lower than for a Southern innovator who has the same unit cost draw.

The ex-ante value of a firm, \( V_{st+1} \), is equal to the present value of the expected future profit flows. The expected rate of return on innovation and imitation must equal the prevailing interest rate in the economy, which is given by:

\[
\mu_{st+1} = \frac{\tilde{\pi}_{st+1}}{V_{st+1}} + \frac{\Delta V_{st+1}}{V_{st+1}},
\]

where \( \Delta V_{st+1}/V_{st+1} \) is the rate of capital gain from a change in the expected firm value. Also, \( V_{st+1} = (M_{st+1}^{I} V_{st+1}^{I} + M_{st+1}^{imit} V_{st+1}^{imit})/M_{st+1} \) for the South.

Potential entrants in the North compare their expected value \( V_{nt+1} \) to \( \eta_{nt+1} \) to decide whether to enter or not. Similarly, potential innovators in the South compare \( V_{st+1}^{I} \) to \( \eta_{st+1}^{I} \). However, there is an indeterminacy for Southern firms to choose between innovation and imitation, given that there is one interest rate to pin down the entry of both innovators and imitators.\(^9\) To pin down \( M_{st+1}^{imit} \) in the South’s household’s problem in Section 2.5, I assume that the expected probability of imitation that a Northern firm faces, \( \phi_{it+1} \), is a function of \( b_s \) and the proportion of Southern innovators among all Southern firms, \( M_{st+1}^{I}/M_{st+1} \). The specific function is given by:

\[
\phi_{it+1} = \nu (1 - b_s) \left( \frac{M_{st+1}^{I}}{M_{st+1}} \right)^{\delta},
\]

where \( \nu > 0 \) and \( \delta > 0 \). The function is increasing in \( M_{st+1}^{I}/M_{st+1} \). Note that the natural way to define the probability of imitation is \( \phi_{it+1} = (1 - b_s) M_{st+1}^{imit}/M_{nt+1} \), the number of surviving Southern imitators as a ratio of Northern innovators. Combining this with (33), \( M_{st+1}^{imit} \) is increasing in \( M_{st+1}^{I}/M_{st+1} \), so that the rate of imitation is positive. Note that if \( \delta < 1 \), the rate of imitation is diminishing as \( M_{st+1}^{I}/M_{st+1} \) increases.

\(^9\)Having two interest rates in the South, i.e. \( \mu_{st+1}^{I} \) and \( \mu_{st+1}^{imit} \), is problematic. \( \mu_{st+1}^{imit} \) must equal to \( \mu_{st+1}^{imit} \) due to arbitrage. However, if all imitators exit the second stage due to uncompetitive \( \gamma \) or \( \tau \), or perfect IPR with \( b_s = 1 \), then \( \mu_{st+1}^{imit} = 0 \). This means that \( \mu_{st+1}^{I} \) will decline to almost 0 right before the time when all imitators exit, and jumps to some positive rate right at the time when all imitators exit. This produces undesirable growth dynamics.
The entry of Northern and Southern innovators are governed by free-entry conditions given by:

\[ V_{nt+1} = \eta_{nt+1}, \quad (34) \]
\[ V_{st+1}^I = \eta_{st+1}^I, \quad (35) \]

The free-entry conditions should hold in principle because if the expected value is greater (smaller) than the cost of R&D, then there will be an infinite (zero) number of firms who are willing to enter, which drives the market interest rate upward (downward) due to excess (zero) demand for R&D resources. Also, as shown in the South’s household problem in below, the choice of \( M_{st+1}/M_{st+1} \) pins down the entry of both innovators and imitators in equilibrium.

Combining the final good sector problem and the two-stage intermediate goods sector problem, economic growth arises from an increasing variety of intermediate goods over time. The growth in the number of intermediate goods is sustainable due to non-decreasing returns to intermediate inputs in final output production. Resources allocated to the R&D sector are increasing in output as long as the investment yields a positive rate of return, thereby maintaining positive R&D growth.

2.5 Representative Households’ Problems

I first rewrite the representative household’s budget constraint to incorporate the aggregation from the second stage problem of the intermediate firms. Since households receive per-period fixed costs payments from intermediate firms in the form of transfers, \( Z_{it} \), the budget constraint (2) can be rewritten as:

\[ B_{it+1} - B_{it} = w_{it}L_i + \mu_{it}B_{it} + N_{dit}F_{dit} + N_{xjt}F_{xjt} - C_{it}. \quad (36) \]

The only investment in the North-South economy is the R&D investment made by intermediate firms. As such, the investment of representative household in \( i \) is equivalent to R&D investment in \( i \), and the household is the firms’ shareholder. I denote \( R_{nt} \) and \( R_{st} \) for Northern and Southern R&D investment, respectively, which are given by:

\[ R_{nt} = \eta_{nt+1} \Delta M_{nt+1}, \quad (37) \]
\[ R_{st} = \eta_{nt+1}^I \Delta M_{st+1}^I + \eta_{st+1}^{imit} \Delta M_{st+1}^{imit}, \quad (38) \]

where \( \Delta \) denotes a change from the last period. R&D investment in the North is equal to the total cost of innovation paid by the new entrants originating from the North, and the South’s investment is equal to the total cost of innovation and imitation paid by the new entrants originating from the South. Assets are accumulated through R&D investments. Assuming that free-entry conditions (34) and (35) hold, and that the firm value of an imitator is equivalent to the cost of imitation, one
can get:\[^{10}\]
\[
B_{nt} = \eta_{nt} M_{nt},
\]
\[
B_{st} = \eta_{st}^l M_{st}^l + \eta_{st}^{imit} M_{st}^{imit}.
\]

Investment income, \(\mu_{it} B_{it}\), consists of the dividends, which are the total net profits of the intermediate goods sector distributed to shareholders, and capital gains earned from the shares of intermediate firms. The Northern and Southern households’ investment income are given by:

\[
\mu_{nt} B_{nt} = N_{nt} \tilde{\pi}_{nt} + \Upsilon_{nt},
\]
\[
\mu_{st} B_{st} = N_{st}^l \tilde{\pi}_{st}^l + N_{st}^{imit} \tilde{\pi}_{st}^{imit} + \Upsilon_{st}^l + \Upsilon_{st}^{imit},
\]

where the \(\Upsilon\) terms denote the total capital gains or losses arising from changes in the total value of intermediate firms originating from country \(i\). Using equations (39) and (40), one can derive that the change in the asset position, \(B_{it+1} - B_{it}\), includes household’s investment (\(R_{it}\)) and capital gains on the asset holdings (\(\Upsilon\)).

From the final good producers’ problem, \(w_{it} L_i\) is equal to \(Y_{it}\) minus the intermediate sector profits, costs of production (\(Q_{it}\)), and fixed costs (\(N_{dit} F_{dit} + N_{xjt} F_{xjt}\)) associated with serving country \(i\).\[^{11}\]

Using optimal prices from Table 1 and average price \(\tilde{p}_{it} = \tilde{a}_{it}/\alpha\), \(\tilde{a}_{it}\) from (27), and \(Q_{it}(p_l)\) from (9), \(Q_{it} = \tilde{a}_{it} N_{it} Q_i(\tilde{a}_{it}) = \alpha^2 Y_{it}\). Together with equations (37)-(42), the budget constraint (36) can be further written as:

\[
C_{it} = (1 - \alpha^2) Y_{it} - E_{it} - R_{it},
\]

where net exports (\(E_{it}\)) is given by:

\[
E_{it} = -[(N_{xit} \tilde{\pi}_{xit} - N_{xjt} \tilde{\pi}_{xjt})].
\]

Note that the negative of E is equal to the net repatriated profits from abroad for country \(i\). Positive net repatriated profits require net imports of the final good (or negative net exports) in order to settle the balance of payments. Similarly, negative net repatriated profits require net exports of the final good.

\[^{10}\]Imposing these assumptions in the household’s problem mean that the optimal policy functions to these problems will yield market-clearing interest rates.

\[^{11}\]Specifically,

\[
q_{it} L_i = Y_{it} - (N_{dit} \tilde{\pi}_{dit} + N_{xjt} \tilde{\pi}_{xjt} + N_{dit} F_{dit} + N_{xjt} F_{xjt} + \tilde{a}_{it} N_{it} Q_i(\tilde{a}_{it})),
\]

where the last term is the total unit cost of production, \(Q_{it}\). Also,

\[
Q_{it} = \tilde{a}_{it} N_{it} Q_i(\tilde{a}_{it})
= \tilde{a}_{it} \alpha^{1-l/(\alpha+1)} a_{it} L_i N_{it} \tilde{a}_{it}^{1/(\alpha+1)}
= \alpha^2 Y_{it}.
\]
I define all the variables in effective terms by lower cases, i.e. for a variable $H_{it}$, define $h_{it} = H_{it}/M_{it}$ for $H_{it} \in \{Y_{it}, C_{it}, R_{it}, E_{it}, N_{it}\}$. For the rest of this section, I drop the time subscript to define the variables used in the Bellman equation using the convention that, for a given variable $h$, $h_{it} = h_i$ and $h_{it+1} = h_i'$. I also define the state $X \in \{M_s^I/M_s, M_s/M_n\}$, in which $M_s^I/M_s \in [0,1]$ given that the number of Southern innovators cannot be larger than the total number of firms originating from the South, and $M_s/M_n > 0$.

The North’s representative household’s value function satisfies the Bellman equation:

$$J_n(X) = \max_{\frac{M_s^I}{M_s}} \left\{ u(c_n(X)) + \beta (1 + g_n(X))^{1-\theta} J_n(X') \right\}$$

s.t.

$$c_n(X) = (1 - \alpha^2) y_n(X) - e_n(X) - g_n(X) \eta_n(X')$$

in which the budget constraint is derived using (37) and (43).

The South’s representative household’s value function satisfies the Bellman equation:

$$J_s(X) = \max_{\frac{M_s^I}{M_s}} \left\{ u(c_s(X)) + \beta (1 + g_s(X))^{1-\theta} J_s(X') \right\}$$

s.t.

$$c_s(X) = (1 - \alpha^2) y_s(X) - e_s(X) - g_s^I(X) \eta_s^I(X') \frac{M_s^I}{M_s} - g_s^{imit}(X) \eta_s^{imit}(X') \left( 1 - \frac{M_s^I}{M_s} \right)$$

which is given by (38) and (43), with

$$g_s^{imit}(X) = \frac{\phi(b_s, M_s^I/M_s')}{\phi(b_s, M_s^I/M_s)} (1 + g_{nt}) - 1$$

derived using (33), taking $g_{nt}$ as given by the North’s household’s problem, and

$$g_s^I(X) = \frac{M_s}{M_s^{imit}} \left( \frac{M_s^I/M_s'}{M_s^I/M_s} - 1 \right) + g_s^{imit}(X)$$

derived from the aggregate rate of technological growth:

$$g_s = \frac{M_s^I}{M_s} g_s^I + \frac{M_s^{imit}}{M_s} g_s^{imit}$$

where $M_s^{imit}/M_s = 1 - M_s^I/M_s$. As seen in (49) and (50), the choice of $M_s^I/M_s'$ pins down $g_s^I$ and $g_s^{imit}$, and hence the entry of innovators and imitators in the South. Recall that there is no population growth in the model. The growth rates of the economy-wide variables are equivalent to their per capita growth rates.

Some restrictions on the parameters $\psi_{sI}$, $\psi_{sI}'$, and $\psi_{sI}^{imit}$ are required to guarantee that the value function $J_n(X)$, and hence $c_n(X)$, are increasing in $M_s/M_n$, and $J_s(X)$, and hence $c_s(X)$, are increasing in $M_s^I/M_s$. First, I divide (26) by $M_s$ so that:

$$y_i = A_i^{\frac{1}{\alpha}} \alpha^{\frac{-2}{\alpha}} L_i n_i a_i^{1-\epsilon}.$$
An increase in \( M_s/M_n \) increases the proportion of exporters among \( n_n \), which in turn increases the average productivity \( \tilde{a}_n^{1-\epsilon} \) as countries are importing inputs with average productivity higher than the domestic inputs, as shown in Melitz (2003). As such, \( y_n \) is increasing in \( M_s/M_n \). On the other hand, an increase in \( M_I^f/M_s \) increases \( n_s \). This is because within \( n_s, n_s^{I,m} \) and \( n_s^{I,c} \) increase one-to-one with \( M_s^I/M_s \), while \( n_s^{imit,m} \) and \( n_s^{imit,c} \) increase by less than one-to-one due to the probability of not being caught by IPR enforcement, given by \( 1 - b_s \), is less than 1 (see (22) to (25)). As such, \( y_s \) is increasing in \( M_I^f/M_s \). Since, \( e_i \) given by (44) divided by \( M_i \), is a function of \( X \) through \( \gamma \) or \( \phi \), and can add or subtract from \( y_i \), restrictions on \( r_i \) are necessary through the parameters \( \psi \) in (28), (29), and (30).

In particular, the upper bounds \( \bar{\psi}_n, \bar{\psi}_s, \) and \( \bar{\psi}_s^{imit} \) set on the \( \psi \)'s guarantee that the \( \eta \) functions (28), (29), and (30) cannot increase excessively as functions of \( M_s/M_n \) or \( M_I^f/M_s \), such that the increase in \( r_i \) is smaller than the increase in \( (1 - \alpha^2)y_i - e_i \). This assumption guarantees that consumption function \( c_i \) is increasing in \( M_I^f/M_s \).

In the next section, I define a recursive equilibrium and the balanced growth path of the model. I then show the existence of an optimal intellectual property rights in the model and discuss how trade policies can help improve welfare given the trade-offs associated with the strengthening of intellectual property rights, and the role of firm heterogeneity which further supports the importance of having a policy mix.

3 Defining Equilibrium and the Balanced Growth Path

Given the initial state variables, \( X_0 \in \{M_I^f/M_s^0, M_s^0/M_n^0\} \), \( M_i^0 \), and \( B_i^0 \) for \( i \in \{n,s\} \), a recursive equilibrium is a set of recursive household policy functions of the state,

\[
X_t \in \{M_I^f/M_s^t, M_s^t/M_n^t\}_{t \in [1,\infty)},
\]

vectors of firm-level variables and cut-off unit costs of intermediate goods sector,

\[
\{V_{nt}, V_{st}^I, V_{st}^{imit}, \eta_{nt}, \eta_{st}^I, \eta_{st}^{imit}\}_{t \in [1,\infty)} \text{ and } \{a_{hit}^{I,m}, a_{hit}^{I,c}, a_{hit}^{imit,m}, a_{hit}^{imit,c}\}_{t \in [1,\infty)} \text{ for } h \in \{d,x\},
\]

vectors of economy-wide sequences and prices,

\[
\{Y_{it}, C_{it}, R_{it}, Q_{it}, E_{it}, B_{it}, \Upsilon_{it}, Z_{it}\}_{t \in [1,\infty)} \text{ and } \{w_{it}, \mu_{it}, \gamma_{it}\}_{t \in [1,\infty)},
\]

such that:

i. Given \( w_{it} \) and \( \mu_{it} \), Northern household’s policy function \( M_s/M_n'(X') \) and value function \( J_n(X) \) solve the Northern household’s problem, and Southern household’s policy function \( M_I^f/M_s'(X') \) and value function \( J_s(X) \) solve the Southern household’s problem;

ii. Final good producers minimize costs by choosing labor and intermediate inputs subject to production function (3), taking prices of intermediate goods as given, and wages satisfy (8);
iii. Intermediate goods firms maximize profit functions by choosing prices, taking into account
the demand function of final good producers (9), the marginal costs of production based on
their unit costs draws, the per-period fixed costs to enter each market, the relative unit cost of
North-South production, $\gamma_t$. $\mu_{it+1}$ satisfies (32);

iv. Free-entry conditions (34) and (35) are satisfied;

v. Markets clear in country $i$, for $i \in \{n, s\}$:

$$L_i = \bar{L}_i,$$

$$B_{it+1} - B_{it} = R_{it} + Y_{it},$$

$$Y_{it} = C_{it} + Q_{it} + R_{it} + E_{it};$$

vi. World markets clear: $E_{nt} = -E_{st}$.

3.1 Balanced Growth Path

Before defining a balanced growth path of the model, I first define a stationary equilibrium as
an equilibrium described above, and in addition, the cut-off costs of intermediate goods sector are
constant, such that the distributions of intermediate firms in the second stage of intermediate firms’
problems are stationary.

The balanced growth path (BGP) is a stationary equilibrium in which all endogenous economy-
wide variables grow at constant rates over time. In particular, the equilibrium growth rates are
the same across the economy-wide variables and across countries. Propositions 1 summarizes this
result. Propositions 2 and 3 summarize the properties of a BGP.

**Proposition 1**: The growth rates of final output, $Y_{it}$, aggregate consumption, $C_{it}$, R&D invest-
ment, $R_{it}$, spending on intermediate goods, $Q_{it}$, net exports, $E_{it}$, assets, $B_{it}$, and the number of
intermediate firms, $M_{it}$, for $i \in \{n, s\}$, are all equal along the BGP and across countries, and are
equal to the BGP rate of technological growth $g^*$.

**Proposition 2**: A BGP exists and is unique, either in the interior or at the corner of $M_n^I/M_s = 1$.

**Proposition 3**: The BGP in the interior is stable.

**Proofs**: See Appendix A.1.

4 Welfare Effects of Strengthening Intellectual Property Rights and Trade
Liberalization

In this section I discuss the welfare results of strengthening IPR in the South and trade liberalization
across both economies. Recall that $b_s$ denotes the exogenous probability of Southern imitator
exiting due to the enforcement of IPR. I assume \( b_s \) as the policy instrument to enforce IPR, in which a higher \( b_s \) represents a strengthening in IPR.

There is a condition which guarantees the South’s growth rate to increase in Southern IPR.\(^{12}\) This is especially important because it separates an emerging market economy that has positive incentive to strengthen IPR from others that do not. Specifically:

**Proposition 4**: Conditional on \( g_s(X) \geq g_n(X) \) for all state \( X \) at a given \( b_{s0} \), and given \( \psi^I_s \geq 0 \) and \( \psi^{imit}_s \geq 0 \) in (29) and (30), respectively, there exists \( \bar{\sigma} \) such that for \( \sigma < \bar{\sigma} \), \( \partial \gamma / \partial (M_{st}/M_{nt}) \) from (11) is sufficiently small \( \forall X \), such that \( \partial g_s(X)/\partial b_s > 0 \forall b_s \in [0,1] \) and \( \forall X \).

*Proof*: See Appendix A.1.

This proposition states that for a South country that is catching up to the North, if the relative cost of production (\( \gamma \)) does not increase at excessively fast pace, then the growth rate in the South is increasing in the level of Southern IPR governed by \( b_s \). The intuition is as follows. When IPR strengthens, the rate of imitation declines. The Southern household needs to decide whether to increase R&D investment in innovation to avoid or minimize the decline in growth. The cost of innovation (\( \eta^I_s(X) \)) is increasing as long as growth is positive. Free entry implies that the firm value (\( V_s(X) \)) also increases. This means that expected profits must increase faster than the increase in firm value in order to raise the expected return on investment in (32), thereby speeding up innovation and the growth rate \( g_s(X) \forall X \). As discussed in Section 2.4, an imitator’s profit must be lower than an innovator’s profit if they have the same unit cost draw due to the probability of IPR enforcement. As a result, an increase in the proportion of innovators in the South increases the expected profits. However, the expected profit (31) also depends on the proportion of firms that can survive in the second stage and make positive profits. An excessively fast increase in the relative cost of production (\( \gamma \)) will shrink the distribution of firms that can survive in the second stage to a degree that can be more than offsetting the increase in profits arising from the increase in innovators’ share. Thus, to guarantee an increase in growth as IPR strengthens, \( \gamma \) cannot increase at excessive speed. Following this result, one can arrive at the following corollary.

**Corollary 1**: The effect of IPR on growth \( g_s(X) \) only depends on the growth in \( \gamma \). As such, it is the growth rather than the level of production costs that matters for determining the effect of strengthening Southern IPR on the South’s growth rate.

Following from Proposition 4, strengthening IPR have a positive consumer welfare effect — the “variety expansion effect”, in which there are consumer welfare gains from increasing home-grown variety due to larger investment in innovation. In particular, increasing innovation increases output

\(^{12}\)If strengthening IPR reduces the South’s growth rate, then one cannot guarantee a positive level of optimal IPR, i.e. \( b_s = 0 \) may be the only optimal IPR.
growth, which in turn increases consumption growth. As explained below, the North’s growth rate also increases, resulting in larger import variety from the North being used in the South’s production. These together mean that there are dynamic gains from increasing variety. However, increase in R&D investment due to strengthening IPR also means more resources are devoted to innovation and less for consumption — the “resource allocation effect”. While the positive variety expansion effect is more than offsetting the negative resource allocation effect before a certain IPR threshold, the negative effect dominates when Southern IPR becomes excessive, thereby reducing the level of consumption particularly along the transition path. The two offsetting effects create an inverted U-shaped relationship between consumer welfare and Southern IPR. As such, there exists an optimal level of Southern IPR from the South’s perspective.

How about from the North’s perspective? The incentive for Northern innovation increases as Southern IPR increases, while import varieties from the South also expands faster given the increase in South’s rate of innovation. These two together expand the growth of production variety in the North and is welfare improving. This is the “variety expansion effect”. On the other hand, the increase in R&D investment takes away resources from consumption — the “resource allocation effect”. An excessive level of IPR means that too much resources will be invested and taken away from consumption, and this can be welfare reducing. Similarly, there exists an inverted U-shaped relationship between North’s consumer welfare and Southern IPR, and there exists an optimal level of Southern IPR from the North’s perspective.

However, the optimal Southern IPR in the North and the South’s perspectives need not be the same given that the economic conditions in the two countries are different. To find one optimal Southern IPR in this economy, one can think of it from the perspective of a planner who weights and maximizes the joint welfare of the two countries. Proposition 5 summarises these results.

**Proposition 5:** For $\sigma < \bar{\sigma}$, there exists an optimal IPR $b_s^* \leq 1$ in which the lifetime consumer welfare in transition and BGP in the South is maximized. There also exists an optimal IPR $b_s^{**} \leq 1$ in which the lifetime consumer welfare in transition and BGP in the North is maximized. However, $b_s^*$ can be different from $b_s^{**}$, and there exists $b_s^{***} \leq 1$ that maximizes the North-South joint welfare.

**Proof:** See Appendix A.1.

Reducing trade costs increases average firm productivity in a heterogeneous firm model as shown in Melitz (2003). As such, one can arrive at the following proposition. Together with Proposition 5, one can formulate Corollary 2 in below.

**Proposition 6:** Consumer welfare in both North and South are strictly increasing in trade liberalization as $\tau$ decreases.

---

13 There is also a third effect in which slower growth in imitated goods decreases the growth in South’s final good production, but again is washed out by the variety expansion effect within the $\beta u'(\bar{c}(X', \zeta(X')))$ term in Euler equation (A.1) in Appendix A.2.
Proof: See Appendix A.1.

**Corollary 2:** When considering the strengthening of IPR in the South, trade liberalization should also be considered at the same time as it may increase the optimal level of IPR.

The idea in this corollary is that, for different $\tau$, there exists a different optimal IPR $b^{***}$. Given that reducing $\tau$ is always welfare improving in the model, the optimal IPR is non-increasing in $\tau$, meaning that reduction in $\tau$ may increase the optimal IPR. Intuitively, the positive effect of trade liberalization through variety expansion due to increased incentive for R&D and increased imports may overcome the resource allocation effect as R&D spending increases along with strengthening IPR up to a certain extent. As such, a policy mix should be considered so as to attain a higher optimal level of IPR.

## 5 Quantitative Analysis

In this section, I quantify the growth and welfare effects of stronger IPR and trade liberalization using data from the US and China. The US data is used for representing the North and the Chinese data for representing the South. Counterfactual experiments based on a calibrated version of the model produce results that echo with the propositions and corollary stated above. Growth and welfare effects along the transition path and BGP will be examined. I first discuss the calibration details in below. The numerical solution method can be found in Appendix A.3.

### 5.1 Calibration

I calibrate the theoretical model to key features of the data around the “initial state”, which I assume it to be around the year 2007, right before the global financial crisis. I assume China’s growth rate to be the 2002-2007 average. This captures the state of the economy right after China’s accession to the World Trade Organization (WTO). The US is assumed to be around its BGP. But because China is on its transitional path, the US economy is also deviating from its BGP, until the two growth rates converge.

Table 2 illustrates the parameter values and their individual calibration targets. The elasticity of substitution ($\epsilon$) of 3.8 comes from Bernard et al. (2003) which is calibrated based on fitting US plant and macro trade data. This implies an intermediate sector’s share of final output of 0.7368 given that $\epsilon = 1/(1 - \alpha)$, which is consistent with the growth literature.\footnote{One can think of intermediate goods as service flows from a broad base of capital, which includes both physical and human capital. In this model, such capital is fully depreciated once it becomes an input of the final good. The intermediate goods sector’s share in the model here can be interpreted as the capital share, which ranges from 0.6 to 0.75 according to Howitt (2000) and Barro and Sala-i-Martin (2004).} The unit cost distribution parameter ($k$) satisfies the slope of the a Pareto distribution of 1.06 for US firm size dispersion as suggested by Axtell (2001) and Luttmer (2007), in which $k/(\epsilon - 1)$ governs the shape
Table 2: Single Parameters Calibrated

<table>
<thead>
<tr>
<th>Params</th>
<th>Descriptions</th>
<th>Values</th>
<th>Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution</td>
<td>3.8</td>
<td>Bernard et al. (2003)</td>
</tr>
<tr>
<td>$k$</td>
<td>Cost distribution</td>
<td>1.06 $\ast$ ($\epsilon - 1$)</td>
<td>Axtell (2001), Luttmer (2007)</td>
</tr>
<tr>
<td>$a_0$</td>
<td>Minimum unit cost draw</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Level of relative unit cost</td>
<td>0.72</td>
<td>China-US average relative unit labor cost of 22.2% in 1998-2003</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of relative unit cost</td>
<td>0.09</td>
<td>2.5% growth in China-US relative unit labor cost in 2002-2007</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Bilateral trade cost</td>
<td>1.42</td>
<td>Novy (2013)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Level of imitation probability</td>
<td>0.02</td>
<td>US import share from China between 2002-2007 after WTO accession</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Elasticity of imitation probability</td>
<td>0.8</td>
<td>11% per capita GDP growth, 2002-2007</td>
</tr>
<tr>
<td>$f_n$</td>
<td>US innovation cost parameter</td>
<td>75</td>
<td>US per capita consumption growth 2.5%</td>
</tr>
<tr>
<td>$f_s$</td>
<td>China innovation cost parameter</td>
<td>180</td>
<td>China’s average patent growth 23%</td>
</tr>
<tr>
<td>$f_{imit_s}$</td>
<td>China imitation cost parameter</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\psi_n$</td>
<td>Elasticity of North innovation cost</td>
<td>-0.01</td>
<td>To maintain 2.5% growth in US</td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>Elasticity of South innovation cost</td>
<td>0.1</td>
<td>Patent growth in China, 2002-2015</td>
</tr>
<tr>
<td>$\psi_{imit_s}$</td>
<td>Elasticity of South imitation cost</td>
<td>0</td>
<td>Assumption to pin down $\nu$ and $\delta$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.98</td>
<td>US real return on stock 7%, 2.5% per capita consumption growth</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Risk aversion</td>
<td>2</td>
<td>Ghironi &amp; Melitz (2005)</td>
</tr>
<tr>
<td>$b_s$</td>
<td>Degree of IPR</td>
<td>0.5</td>
<td>Between 0 and 1</td>
</tr>
<tr>
<td>$M_{s0}/M_{d0}$</td>
<td>Initial state variable</td>
<td>0.2</td>
<td>Small starting value, but sufficiently &gt; 0</td>
</tr>
<tr>
<td>$L_n$</td>
<td>US labor force</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$L_s$</td>
<td>China’s labor force</td>
<td>4.3</td>
<td>China-US population ratio, 1960-2015</td>
</tr>
</tbody>
</table>

of firm revenue distribution in the model here. I normalize the minimum possible value of unit costs ($a_0$) to be 1. The level of relative unit cost ($\lambda$) between the North and the South is assumed to be 0.72, and the elasticity of the relative unit cost ($\sigma$) is 0.09. These two together yields an average relative unit labor cost of 22.2% between China and US for the initial period using $\gamma$ from (11), and at an average growth rate of 2.5% afterwards. The unit labor cost estimate for the initial period is taken from Ceglowski and Golub (2011)’s estimate based on World Bank and UNIDO data, and the growth rate is my own estimate from the relative unit labor cost constructed using OECD and China’s National Bureau of Statistics data. As suggested by Novy (2013), the iceberg trade cost ($\tau$) of 1.42 among OECD countries is the estimated trade-weighted average of US bilateral trade costs as of 2000.

The parameter $\nu$ governs the level of imitation probability, and the initial probability is equal to the US import share from China. This serves to capture a more realistic impact of Chinese imitation on US through the trade channel. The elasticity of imitation probability ($\delta$) governs the speed of imitation growth in China, as discussed in Section 2.4. Since the South’s output growth is an increasing function of domestic imitation and innovation growth, and decreasing in the relative cost of production, $\delta$ is calibrated to yield an 11% per capital output growth in the initial period
given the assumption of an initial innovation rate of 23%, which is China’s average patent growth between 2002 and 2015, and a 2.5% increase in $\gamma - 1$ which is subtracted from the 11% growth.\textsuperscript{15,16}

The level of US cost of innovation ($f_n$) is calibrated to match the long-run average of US per capita consumption growth. The level of China’s cost of innovation ($f_s^I$) is calibrated to match with China’s average patent growth. China’s imitation cost parameter ($f_s^{imit}$) is normalized to be 1. The elasticity of North innovation cost ($\psi_n$) is calibrated to slightly negative, so as to maintain US growth rate at slightly above 2.5% in the long run.\textsuperscript{17} The elasticity of South innovation cost ($\psi_s^I$) is calibrated to match the decline in patent growth in China between 2002 and 2015. This is also consistent with Bollard, Klenow, and Li (2016) which show empirical evidence on rising entry costs along with development using data on US, India, and China in recent decades. The elasticity of South imitation cost ($\psi_s^{imit}$) is assumed to be zero for simplicity, given that the imitation probability has already been identified in the model. Also, it works out that the cost of imitation is lower than the cost of innovation in the South, consistent with findings in Mansfield, Schwartz, and Wagner (1981).

The discount factor ($\beta$) is calibrated using the consumption Euler equation to match the US data on its long-run real return on stock and per capita consumption growth. The risk aversion parameter ($\theta$) is chosen to be 2. The degree of IPR ($b_s$) is assumed to be in between 0 and 1 given that it is a probability of Southern imitators being caught for IPR infringement. I choose it to be 0.5. I choose the initial state variable $M_{s0}/M_{n0}$ to be small in order to capture transitional dynamics, but sufficiently larger than 0 based on experimenting with the numerical results. Finally, I normalize US labor force ($L_n$) to be 1, and assume China’s labor force ($L_s$) to be equal to the China-US population ratio. This helps to calculate results in per capita terms.

Table 3 describes the facts that are used to jointly calibrate the remaining parameters. I normalized the North’s aggregate productivity parameter ($A_n$), initial number of Northern intermediate firms ($M_{n0}$), and domestic fixed cost parameter ($f_{dn}$) to be 1, and calibrated the South’s aggregate productivity parameter ($A_s$), the initial state variable $M_{s0}/M_{n0}$, South’s domestic fixed cost parameter ($f_{ds}$), and the exporters’ fixed costs parameters ($f_{xn}$ and $f_{xs}$) jointly to match the China-US per capita GDP ratio and the average export-to-GDP ratios in the US and China for the period.

\textsuperscript{15}Assuming the sum of the patents per year before 2002 as the stock of patents cumulated, I calculate the average annual growth rate of patent counts by Chinese companies between 2002 and 2015. I used three sets of data to cross-check the reliability: the “invention” patent counts of domestic origin from China’s State Intellectual Property Office (SIPO), the “utility” patent counts of Chinese origin in the US using data from US Patent and Trademark Office (USPTO), and the data from OECD triadic patent database on Chinese origin patents filed at the European Patent Office (EPO), the Japanese Patent Office (JPO), and the USPTO. The three sets of data yield 23-26% annual growth in patent counts. I take the lower value for the calibration exercise.

\textsuperscript{16}Given the assumption that $M_{s0}/M_{n0} = 0.2$, a patent growth of 23% means that innovation growth contributes to the South’s initial output growth by 4.6 percentage points, and with a 2.5 percentage points increase in the average relative unit cost, imitation growth contributes by 6.4 percentage points.

\textsuperscript{17}It turns out that if $\psi_n$ is to zero or slightly positive, US consumption growth rate will decline from 2.5% to 2.2% over time in the numerical solution to the North’s problem.
Table 3: Multiple Parameters Calibrated

<table>
<thead>
<tr>
<th>Params</th>
<th>Descriptions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_n$</td>
<td>Aggregate productivity (normalization)</td>
<td>1</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Aggregate productivity</td>
<td>0.86</td>
</tr>
<tr>
<td>$M_{n0}/M_{s0}$</td>
<td>Initial state variable</td>
<td>0.1</td>
</tr>
<tr>
<td>$M_{n0}$</td>
<td>Initial no. of Northern firms (normalization)</td>
<td>1</td>
</tr>
<tr>
<td>$f_{dn}$</td>
<td>Fixed costs (normalization)</td>
<td>1</td>
</tr>
<tr>
<td>$f_{ds}$</td>
<td>Fixed costs</td>
<td>5</td>
</tr>
<tr>
<td>$f_{xi}$</td>
<td>Fixed costs, $i \in {n, s}$</td>
<td>$&gt;800$</td>
</tr>
</tbody>
</table>

Targets

| China-US per capita GDP ratio | 0.14 |
| China export share of GDP (2002-2007) | 10%  |
| US export share of GDP (2002-2007) | 0.3% |

Table 4: Other Quantitative Implications

<table>
<thead>
<tr>
<th>Model</th>
<th>Model Implied</th>
<th>Sources</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markup over marginal cost</td>
<td>12.8%</td>
<td>Basu &amp; Fernald (1997)</td>
<td>8-19%</td>
</tr>
<tr>
<td>North R&amp;D share of output</td>
<td>6.0%</td>
<td>OECD (highest)</td>
<td>3.9%</td>
</tr>
<tr>
<td>South R&amp;D share of output</td>
<td>8.7%</td>
<td>OECD (highest)</td>
<td>3.9%</td>
</tr>
<tr>
<td>North innovation cost (per Y/L)</td>
<td>2.4</td>
<td>Barseghyan &amp; DiCecio (2011)</td>
<td>2.3-15.2</td>
</tr>
<tr>
<td>South innovation cost (per Y/L)</td>
<td>8.0</td>
<td>Barseghyan &amp; DiCecio (2011)</td>
<td>2.3-15.2</td>
</tr>
<tr>
<td>South imitation cost (per Y/L)</td>
<td>0.1</td>
<td>Barseghyan &amp; DiCecio (2011)</td>
<td>2.3-15.2</td>
</tr>
</tbody>
</table>

Import shares (% final output)

<table>
<thead>
<tr>
<th>Model</th>
<th>Values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>US import share</td>
<td>2.1%</td>
<td>OECD STAN</td>
</tr>
<tr>
<td>China import share</td>
<td>1.4%</td>
<td>OECD STAN</td>
</tr>
</tbody>
</table>

Fixed costs (per-period) values

<table>
<thead>
<tr>
<th>Model</th>
<th>Values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Domestic</td>
<td>$1,490</td>
<td>$1,900-7,500</td>
</tr>
<tr>
<td>China Domestic</td>
<td>$2,923</td>
<td>$1,900-7,500</td>
</tr>
</tbody>
</table>

between 2002-2007. The relevance of welfare measures can be improved as a result of matching the relative per capita GDP ratio, the relative population size, and the export shares. All GDP and consumption data in the calibration exercise come from World Bank’s World Development Indicator, and the trade data come from the OECD STAN data set.

Table 4 illustrates other quantitative implications that are generated by the parameterized version of the model, but were not targeted as part of the calibration exercise. First, the elasticity of substitution ($\epsilon$) of 3.8 would yield a 12.8% markup over marginal cost in the model, which is consistent with those in the macroeconomic literature such as Basu and Fernald (1997). The model

\[ Y_{it} - Q_{it} \] for $i \in \{n, s\}$ because the share of final output used as intermediate inputs is not value-added. GNP is defined as $Y_{it} - Q_{it} - E_{it}$ to account for the net repatriated profits from abroad. I calibrate the exporters fixed costs to match the export-to-final output ratios in the model with the export-to-GDP ratios in the data.

\[ \text{Note that GDP in the theoretical model is } Y_{it} - Q_{it} \text{ for } i \in \{n, s\} \text{ because the share of final output used as intermediate inputs is not value-added. GNP is defined as } Y_{it} - Q_{it} - E_{it} \text{ to account for the net repatriated profits from abroad. I calibrate the exporters fixed costs to match the export-to-final output ratios in the model with the export-to-GDP ratios in the data.} \]
yields a rather high R&D share of output compared with the shares among OECD countries, but arguably they are not implausibly large. The costs of innovations, expressed as a share of per capita final output, are consistent with the entry costs of US firms in Barseghyan and DiCecio (2011). Even though the entry costs may not be equivalent to the average cost of R&D at the firm level, it is still a good indication that the magnitude of firms’ entry costs based on model calibration here are not unreasonable. The middle panel of Table 4 shows that the magnitudes of US and China import shares are similar to the actual shares found in the data. The bottom panel shows that the dollar value of the US per-period domestic fixed costs implied by the model is somewhat lower than the range estimated in Rodrigue (2014) based on Indonesian plant-level data, whereas the model-implied per-period domestic fixed cost in China is within the estimated range.19

5.2 Counterfactual Experiments

I conduct counterfactual experiments to examine the growth and welfare effects of trade liberalization and strengthening of intellectual property rights. Specifically, I first solve numerically the North and the South’s problems using the calibrated version of the model. I call this the “benchmark case”. The benchmark $\tau = 1.42$ and the benchmark $b_s = 0.5$. Next, I solve for the model with different iceberg trade costs ($\tau$) and/or IPR ($b_s$) at time 0, for a scheme of bilateral iceberg trade costs $\tau \in \{1.74,1.546,1.294,1.21,1.1\}$, and a scheme of IPR $b_s \in \{0.2,0.4,0.55,0.6,0.65,0.7,0.75,0.8\}$, holding all other calibrated parameters constant. The reasons for choosing these $\tau$ are as follows: 1.74 is the historical level of iceberg trade cost in 1970 according to Novy (2013); 1.546 is a 30% increase from the benchmark; 1.294 is a 30% reduction from the benchmark $\tau$; and 1.1 is a reduction with the same magnitude as the increase in $\tau$ to the historical level from the benchmark $\tau$. I compute the growth rates in the counterfactual cases and the welfare gains/losses in effective consumption terms relative to that in the benchmark case.

5.2.1 Trade Liberalization

Trade liberalization generally increases the US rate of innovation (the growth rate of the number of Northern innovators in the model). As shown in Figure 1.1, the time path of innovation rate shifts up as $\tau$ decreases from the benchmark of 1.42 all the way to 1.1. But the effect is asymmetric for the cases when trade restrictions are imposed. While increasing $\tau$ from 1.42 to 1.546 shifts the growth path downward, further increasing $\tau$ to 1.74 actually increases growth above the benchmark growth path after $t = 7$. As seen below, the welfare loss from increasing $\tau$ to 1.74 is disproportionately larger than by increasing it to 1.546. Northern household wants to invest in innovation to mitigate the welfare loss which would otherwise be even bigger. This causes the rate of innovation to be

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19The fixed costs values were originally expressed in 1983 US dollars in Rodrigue (2014). These values are converted to 2010 US dollars for consistency with the data used in the calibration exercise here.
higher for the latter period.\textsuperscript{20} That said, one should be reminded that the distance of curve shifting is quite minimal, with the distance between the highest and lowest curves in the figure being no more than 0.02 percentage point.

The time paths of US growth rate are upward sloping given that I allow for a small technology spillover due to decreasing cost of innovation ($\eta_n$) to maintain a relatively flat shape of the growth rate schedule at larger state $X$ at the later periods. The flat tail and small shifting of the growth path are consistent with the assumption that the US is close to the BGP.

Figure 1.2 shows that China’s rates of R&D growth are generally decreasing over time. The rate of R&D growth refers to the weighted average of the rates of innovation and imitation given by equation (51). The downward sloping time path is mostly due to the diminishing return on

\textsuperscript{20}While an increase in $\tau$ can shift aggregate R&D schedule downward for a given state $X$, it also raises the slope of the R&D schedule given that the schedule is increasing and concave, and hence a higher rate of innovation.
having innovation growth in excess of the BGP growth rate. The slower increase in the probability of imitation (33) and therefore the decline in the rate of imitation over time also has a role to play in reducing the speed of R&D growth, but this is not the main driver here.\textsuperscript{21} To take a closer look at China’s growth paths, Figure 1.3 shows that in the first 10 periods, the time path of China’s rate of R&D growth shifts upward monotonically when \( \tau \) decreases, which is different from the US case.

To better evaluate the effect of trade liberalization, I conduct a welfare analysis on both the equilibrium transition path and the BGP for both the US and China. To obtain a measure of welfare changes along the equilibrium transition path, I solve for \( \Delta c_i \) for \( i \in \{n, s\} \) in the following equation:

\[
\bar{U}_{\text{trans}}^i = \sum_{t=0}^{T-1} \beta^t M^{0,1-\theta}_it(c^{0}_it + \Delta c^{\text{trans}}_i)^{1-\theta} \frac{1}{1 - \theta}
\]

where \( \bar{U}_{\text{trans}}^i \) is the level of lifetime utility attained on the equilibrium transition path for a given scenario. \( M^{0}_it \) and \( c^{0}_it \) are country \( i \)’s number of intermediate firms and effective consumption at time \( t \) under the equilibrium transition path in the benchmark case, where superscript 0 denotes “benchmark”. \( \Delta c_i \) denotes the change in effective consumption that compensates the benchmark effective consumption to attain \( \bar{U}_{\text{trans}}^i \). The welfare measure accounts for all welfare gains/losses between the initial period up to time \( T - 1 \), in which \( T \) is the first period when the BGP is attained either under the benchmark equilibrium path or the counterfactual equilibrium path, whichever is longer. The transitional welfare gain is expressed as \( \frac{\Delta c^{\text{trans}}_i}{c^{0}_i} \times 100 \), where \( c^{0}_i \) is the initial period’s effective consumption under the benchmark case.

To obtain a measure of welfare changes along the BGP, I compute welfare as if the economy starts off along the BGP at \( t = 0 \) and will stay along the BGP forever. I solve for \( \Delta c^{\text{bgp}}_i \) in the following equation:

\[
\bar{U}_{\text{bgp}}^i = \frac{1}{1 - \beta(1 + g^*)^{1-\theta}} \frac{(c^{0,\text{bgp}}_i + \Delta c^{\text{bgp}}_i)^{1-\theta} - 1}{1 - \theta}
\]

where \( \bar{U}_{\text{bgp}}^i \) is the level of lifetime utility attained along the BGP for a given scenario. \( g^* \) is the BGP growth rate in the benchmark case. \( c^{0,\text{bgp}}_i \) is the benchmark effective consumption along the BGP, which is constant. \( \Delta c^{\text{bgp}}_i \) is the change in effective consumption that compensates the benchmark effective consumption to attain \( \bar{U}_{\text{bgp}}^i \). The BGP welfare gain is expressed as \( \frac{\Delta c^{\text{bgp}}_i}{c^{0}_i} \times 100 \). Here I assume \( M_{i0} = 1 \) for normalization, and it is growing at \( g^* \) along the BGP. Hence, the discount factor includes the \((1 + g^*)^{1-\theta}\) term.

The overall welfare gain is given by \((\beta^T \Delta c^{\text{bgp}}_i + (1 - \beta^T) \Delta c_i)/c^{0}_i \times 100\), which is the weighted sum of BGP and transitional welfare gains, evaluated against the benchmark effective consumption at the initial period. The weight \( \beta^T \) reflects the importance of BGP welfare gains for today. Since

\textsuperscript{21}From the calibration exercise, China’s initial rate of innovation is about 23%, whereas the initial rate of imitation is about 11%. As the rates of innovation and imitation will decline to the BGP growth rate of around 2.5-2.6% over time, the reduction and hence the slope of innovate rate must be steeper than the slope of the imitation rate.
Table 5: Trade restrictions and liberalization — welfare gain (%)

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>Trans</th>
<th>BGP</th>
<th>Overall</th>
<th>Trans</th>
<th>BGP</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.74 (hist)</td>
<td>-6.84</td>
<td>-9.38</td>
<td>-7.47</td>
<td>-2.76</td>
<td>-1.80</td>
<td>-2.52</td>
</tr>
<tr>
<td>1.546 (+30%)</td>
<td>-0.22</td>
<td>-0.78</td>
<td>-0.35</td>
<td>-1.36</td>
<td>-1.25</td>
<td>-1.33</td>
</tr>
<tr>
<td>1.42 (benchmark)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1.294 (-30%)</td>
<td>0.25</td>
<td>0.56</td>
<td>0.32</td>
<td>1.76</td>
<td>1.34</td>
<td>1.66</td>
</tr>
<tr>
<td>1.21 (-50%)</td>
<td>0.77</td>
<td>1.30</td>
<td>0.90</td>
<td>3.33</td>
<td>2.30</td>
<td>3.07</td>
</tr>
<tr>
<td>1.1 (-hist %)</td>
<td>3.56</td>
<td>4.42</td>
<td>3.78</td>
<td>6.16</td>
<td>3.69</td>
<td>5.55</td>
</tr>
</tbody>
</table>

there is no population growth in the model, the aggregate and per capita welfare gains/losses are the same.

Table 5 shows the transitional, BGP, and overall welfare effects for the US and China under the various cases of $\tau$. Under the “Trans” column, the transitional welfare gains increase for both the US and China as $\tau$ decreases from the benchmark, and the welfare losses increase as $\tau$ increases from the benchmark. For instance, by reducing $\tau$ to 1.1, which is equivalent to a 76% iceberg trade cost reduction, the US’ transitional gain is equivalent to 3.56% of today’s effective US consumption, and China’s transitional gain is equivalent to 6.16% of today’s effective consumption in China. By increasing $\tau$ to the 1970 cost of 1.74, US suffers a transitional welfare loss of 6.84%, which is larger than the 2.76% welfare loss in China. As for the BGP welfare effects, the magnitudes of gains and losses for the US are bigger than along the transition paths for all $\tau$’s, whereas the BGP welfare effects are smaller than in transition for China.

Looking at the overall welfare gains, we can see that the overall welfare effects exhibit two patterns: (i) the overall welfare is strictly decreasing in $\tau$, and (ii) the overall welfare loss for the US is bigger than for China when $\tau = 1.74$, but for all other cases of $\tau$, the total welfare effects for the US are smaller than those for China. Result (i) resembles the theoretical result stated in Proposition 6. To explain result (ii), recall that in Figure 1.1, the time path of US rate of innovation for $\tau = 1.74$ is higher than under the benchmark case after several periods into transition. Even though the growth effect is positive in this case, it is sacrificing too much consumption in level terms in both transition and BGP.

5.2.2 Strengthening Intellectual Property Rights

In this section, I show that there exist optimal levels of IPR from the North and the South perspectives which are different, and we can find an optimal IPR by evaluating the joint welfare of North and South. In addition, I show that it is welfare-improving to consider a policy mix of stronger IPR and trade liberalization.

The time paths of US rate of innovation and China’s rate of R&D growth shift up as the degree of IPR increases above the benchmark, and vice-versa, which are illustrated in Figures 2.1 and 28.
2.2. First, by decreasing $b_s$ from 0.5 to 0.2, the time paths are flat for both US and China. It is a corner solution in which the US-China economy jumps to the new BGP at the moment when the South’s IPR changes.\footnote{Indeed, the welfare losses for reducing IPR from 0.5 to 0.2 are large. For the US, the overall welfare loss is 3.3% in effective consumption terms. For China, the loss is 74%. The big welfare loss is due to the drop in China’s R&D growth from a double-digit rate during the initial periods along the benchmark transition path down to only a constant 2.5% for all periods along the alternative path. The variety expansion effect is strong given that the effective cost of imitation after accounting for the probability of IPR enforcement is cheap when IPR is weak. As such, investment in innovation drops to zero. South’s household chooses $M_s^I/M_s$ to stay constant, and imitators copy at the same rate as North’s rate of innovation. As for the North, the welfare loss is due to a decline in incentive to innovate as expected profits decline, given that more Northern firms exit as more ideas are being copied. The decrease in R&D growth rate in the South also means decreased import variety expansion in the North. For the less extreme case in which IPR is reduced from 0.5 to 0.4, the US and China’s overall welfare losses are 1.3% and 15.7%, respectively, again due to the dynamic effects of slower growth on the consumption paths.} Focusing on the cases when IPR ($b_s$) is raised above 0.5, the upward shifts in the time paths mean positive variety expansion effects of welfare gains. However, there is also a negative resource allocation effect which reduces effective consumption due to increased R&D spending when IPR strengthens. As discussed in Section 4, the two effects together create an inverted U-shaped relationship between consumer welfare and IPR.

Table 6 shows that there are optimal levels of IPR from the North and the South’s perspectives, but the two optimal levels are different. First, looking at the “Trans” columns, US transitional welfare gain increases as $b_s$ increases from 0.5 to 0.55, but the gain starts to decline as IPR increases further and turns into a welfare loss at $b_s = 0.65$ and onward. For China, the transitional welfare gain increases as $b_s$ increases to 0.55 and further to 0.6, and slowly decline afterwards and eventually turns negative at $b_s = 0.8$. On the other hand, BGP gains are always increasing in IPR for both US and China.

Due to the inverted U-shaped relationship between transitional welfare and IPR, the overall welfare gains also exhibit an inverted U-shaped relationship. The optimal IPR is at $b_s = 0.6$ from...
Table 6: IPR strengthening when \( \tau = 1.42 \) — welfare gains (%)

<table>
<thead>
<tr>
<th>IPR</th>
<th>US Trans</th>
<th>US BGP</th>
<th>US Overall</th>
<th>China Trans</th>
<th>China BGP</th>
<th>China Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.55</td>
<td><strong>0.12</strong></td>
<td>0.88</td>
<td>0.29</td>
<td>4.98</td>
<td>2.01</td>
<td>4.31</td>
</tr>
<tr>
<td>0.6</td>
<td>0.08</td>
<td>1.55</td>
<td><strong>0.38</strong></td>
<td><strong>7.37</strong></td>
<td>3.61</td>
<td>6.61</td>
</tr>
<tr>
<td>0.65</td>
<td>-0.10</td>
<td>2.06</td>
<td>0.32</td>
<td>7.37</td>
<td>4.89</td>
<td><strong>6.89</strong></td>
</tr>
<tr>
<td>0.7</td>
<td>-0.38</td>
<td>2.41</td>
<td>0.11</td>
<td>5.23</td>
<td>5.94</td>
<td>5.35</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.78</td>
<td>2.66</td>
<td>-0.22</td>
<td>1.08</td>
<td>6.75</td>
<td>2.00</td>
</tr>
<tr>
<td>0.8</td>
<td>-1.27</td>
<td>2.79</td>
<td>-0.65</td>
<td>-4.93</td>
<td>7.38</td>
<td>-3.05</td>
</tr>
</tbody>
</table>

Note: Red color indicates optimal.

Table 7: Alternative measure of overall welfare gains (%)

<table>
<thead>
<tr>
<th>IPR</th>
<th>US</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.55</td>
<td>0.50</td>
<td>3.49</td>
</tr>
<tr>
<td>0.6</td>
<td>0.82</td>
<td>5.49</td>
</tr>
<tr>
<td>0.65</td>
<td>0.98</td>
<td><strong>6.13</strong></td>
</tr>
<tr>
<td>0.7</td>
<td><strong>1.01</strong></td>
<td>5.58</td>
</tr>
<tr>
<td>0.75</td>
<td>0.94</td>
<td>3.92</td>
</tr>
<tr>
<td>0.8</td>
<td>0.76</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Note: The overall welfare measure here weights the transitional and BGP welfare gains by half-half.

the North’s perspective, and at \( b_s = 0.65 \) from the South’s perspective. By imposing a different weights, say half-half on transitional and welfare gains (instead of \( \beta^T \)), the overall welfare gains still exhibit an inverted U-shaped relationship as shown in Table 7. Thus, optimal IPR also exists according to this alternative overall welfare measure. This result is consistent with Proposition 5. Indeed, the statement is general enough and is applicable to a wide range of weightings on transitional and BGP welfare that produce the overall welfare measures, unless one cares much more about the BGP welfare than the transitional welfare by giving a sufficiently small weighting on the transitional welfare.

Given that trade liberalization is always welfare-improving in this model, it should be considered together with the strengthening of IPR in the South, as the policy mix may increase the optimal level of IPR as stated in the Corollary 2. In Figures 3.1 and 3.2, I plot the time paths of US innovation rate and China rate of R&D growth for the various IPR levels at \( \tau = 1.1 \). Generally speaking, the time path for every IPR level is higher when \( \tau = 1.1 \) than in Figures 2.1 and 2.2 when \( \tau = 1.42 \). Table 8 shows the welfare gains when \( \tau = 1.1 \), comparing to the benchmark case of \( \tau = 1.42 \) and \( b_s = 0.5 \). The transitional, BGP, and overall welfare gains are all higher than their
Figure 3: Growth effects of IPR when $\tau = 1.1$

<table>
<thead>
<tr>
<th>IP R</th>
<th>Trans US</th>
<th>Trans China</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BGP</td>
<td>Overall</td>
</tr>
<tr>
<td>0.5 ($\tau = 1.42$)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>3.56</td>
<td>4.42</td>
</tr>
<tr>
<td>0.55</td>
<td>3.98</td>
<td>6.20</td>
</tr>
<tr>
<td>0.6</td>
<td>4.28</td>
<td>7.94</td>
</tr>
<tr>
<td>0.65</td>
<td>4.43</td>
<td>9.61</td>
</tr>
<tr>
<td>0.7</td>
<td>4.43</td>
<td>11.18</td>
</tr>
<tr>
<td>0.75</td>
<td>4.31</td>
<td>12.81</td>
</tr>
<tr>
<td>0.8</td>
<td>4.05</td>
<td>14.41</td>
</tr>
</tbody>
</table>

Table 8: IPR strengthening when $\tau = 1.1$ — welfare gains (%)  

Respective counterparts in Table 6. Also, the optimal level of IPR for the US is $b_s = 0.75$ according to the overall welfare measure, which is higher than $b_s = 0.6$ when $\tau = 1.42$. However, the optimal level of IPR for China stays the same at $b_s = 0.65$.

As there are different optimal levels of IPR for the North and the South, it is possible to find a joint optimal IPR for the North-South economy. Assuming that there is a planner who weights the welfare of the two countries and decides the optimal IPR based on the joint welfare gains. Here, I take the weights as given rather than trying to find the optimal weightings. Table 9 shows the joint welfare gains by assigning weights of 0.5/0.5, 0.7/0.3, and 0.9/0.1 on the US and China’s overall welfare gains from Table 6 (for $\tau = 1.42$) and Table 8 (for $\tau = 1.1$). It turns out that the joint optimal IPR is at $b_s = 0.65$ for both $\tau$ when the weightings are 0.5/0.5 and 0.7/0.3, which also coincides with the optimal IPR from China’s perspective. This is due to the larger gains of stronger Southern IPR for China than for the US.

However, when the weights are shifted to 0.9/0.1, i.e. when the planner weights heavily on the
US than on China, the joint optimal IPR changes from 0.6 to 0.7 as τ decreases, which is close to the optimal IPR of 0.75 from the US perspective. This is because the effect of trade liberalization relative to stronger Southern IPR from the US perspective is larger than the relative importance for China. To illustrate this point, from Table 6, by increasing Southern IPR to the optimal level, China’s welfare gain is 7%, and is 13% together with trade liberalization as shown in Table 8. The respective welfare gains for the US are 0.4% and 6%. The trade openness and IPR policy mix yields a US welfare gain that is 15 times of that when there is a strengthening in Southern IPR alone, whereas China’s has doubled. Not only that trade liberalization can possibly raise the level of optimal IPR, it can also affect the distribution of welfare gains between countries. This points to the importance for policymakers to consider trade and IPR policies together.

5.2.3 The Role of Firm Heterogeneity

Finally, I would like to highlight the role of firm heterogeneity through which strengthening IPR can affect the average productivity of technology used for production in country i. Due to firm heterogeneity, strengthening IPR can affect firm survival in the second stage, which in turn can affect aggregate firm productivity from the production perspective, as well as firm’s expected profits from the R&D perspective. Here, I focus on the production perspective. Average firm productivity is given by $\tilde{a}_{it}^{1-\epsilon}$, in which $\tilde{a}_{it}$ is given by (27).

Figures 4.1 and 4.2 illustrate the average firm productivity of Northern and Southern firms, respectively, in (i) the benchmark case with $b_s = 0.5$, (ii) the case when τ = 1.42 with the joint optimal IPR of $b_s = 0.65$, and (iii) the case when τ = 1.1 with the joint optimal IPR of $b_s = 0.65$. The North (US) average firm productivity increases over time because of the increasing proportion of imports from the South used in production, in which the imported inputs are on average more productive than the domestic inputs, which is a feature of the Melitz model. On the other hand, due to the differential between the North and the South’s rates of technology creation, the proportion
Figure 4: Average firm productivity

Note: Average firm productivities are normalized by the respective average firm productivity in the benchmark case at \( t = 1 \).

of the South (China) imports from the North decreases, thus the South’s average firm productivity decreases over time.

Given \( \tau = 1.42 \), the strengthening in IPR reduces US average firm productivity initially compared with the benchmark case, but it rises above the benchmark productivity level for the latter periods. This is because when IPR strengthens, there are less imported immediate inputs from Southern imitators, which reduces the US average firm productivity. This is eventually reversed by increasing imported immediate inputs from Southern innovators due to the faster rate of innovation in the South.

On the other hand, China’s average firm productivity increases initially but drops below the benchmark productivity afterwards. The initial rise has to do with the initial increase in imports of Northern immediate goods as the probability of imitation faced by Northern firms decline. But because of the pick up in the growth differential between the South and the North after a strengthening in IPR, the proportion of Northern imports gradually decreases compared with that in the benchmark case, thereby reducing China’s average firm productivity in the latter period.

When \( \tau \) is sufficiently reduced, average firm productivity increases in both countries due to increase in imported technology. While a strengthening in IPR by itself may benefit the average firm productivity in the North but reduce it in the South, a policy mix with trade openness through reducing tariffs and non-tariff barriers can improve firm productivity in both countries and the optimal level of Southern IPR.
6 Conclusion

This paper studies the optimal level of intellectual property rights (IPR) in a large emerging market economy, both from an advanced economy and the large emerging economy’s perspectives. I present a North-South model of endogenous technological change with firm heterogeneity, in which growth is driven by variety expansion, with innovation and imitation co-exist in the South. In the theoretical results, I show that it is the growth rather than the level of production costs that matters for determining the effect of strengthening Southern IPR on the South’s growth rate. The effect is positive if the growth of production costs is not excessively fast. Following this result, I show that there exists an inverted U-shaped relationship between consumer welfare and Southern IPR for both the North and the South, arising from the trade-off between a variety expansion effect and a resource allocation effect due to increased R&D investment at the expense of effective consumption. Also, the optimal Southern IPR from the North and the South’s perspectives can be different due to different economic conditions.

I conduct quantitative analysis to examine the growth and welfare effects of stronger IPR and trade liberalization. I use the US data to represent the North and the Chinese data to represent the South. Counterfactual experiments show that by increasing Southern IPR to the optimal level, China’s welfare gain is 7%, and can be up to 13% together with trade liberalization. However, given that the US consumer welfare is more sensitive to trade openness than to Southern IPR, US welfare gain is only 0.4% if Southern IPR is raised to the optimal level, but together with trade liberalization, welfare gain increases to 6% . This is a 15 times increase, compared with China’s which is doubled. Not only that trade openness can potentially raise the optimal IPR, it can also affect the distribution of welfare gains between countries.

In addition, strengthening IPR raises US average firm productivity through increasing import variety, but reduces it in China due to less import variety. I show that there is room for further trade openness to improve average firm productivity, which supports the importance of having a policy mix of trade openness and IPR that can be welfare-improving.

The policy implication goes beyond such a policy mix. Given this paper’s results that the speed of production cost growth determines the effectiveness of strengthening IPR, regardless of the low level of production costs in the South, emerging market economies that face rapid increase in production costs should implement reforms and measures to alleviate cost pressures, such as tax and labor market reforms, as well as infrastructure investment that can benefit general firm productivity.

While this model is suitable for studying the interaction between advanced economies and large emerging market economies such as China or India, it may not be the case for studying a South economic region that aggregates many small emerging market economies into a single economy. The behavior of small emerging economies can be very different from large emerging economies when it
comes to the choice between innovation and imitation, as the resources available for R&D activities and the returns from leapfrogging can be much limited compared to a large emerging economy. As such, grouping small emerging economies and treating as one big economy seems inappropriate. The transitional welfare effects of intellectual property rights between large and small economies is still an open question. It can be addressed by extending the quantitative model presented in this paper to a multi-country setting.
Appendix A

A.1 Proofs of Propositions

Proof of Proposition 1: Define $g^*$ as the BGP rate of technological growth. Along the BGP, $J_i(X) = J_i(X')$ for $i \in \{n, s\}$ in (45) and (47). This happens if and only if $X' = X$, which further implies that $g^*_s = g^*_s$ and $g_s = g_n = g^*$, and that $\eta_n(X'), \eta_s(X')$ and $\eta^*_s(X')$ are constant. Given that the intermediate firms’ distributions are stationary, $y_i$ and $e_i$ are constant in (46) and (48). These altogether imply that $c_i$ in (46) and (48) are constant. As the economy-wide variables in $i$ are constant in effective terms (i.e. as a ratio of $M_i$), these variables in aggregate terms grow at rate $g_i$. Since $g_s = g_n = g^*$, all economy-wide variables grow at the same rate along the BGP and across countries, which are also equal to $g^*$.

Proof of Proposition 2: Given the restrictions imposed on $\psi^i_s$ and $\psi^*_{init}$, consumption $c_s(X)$ is increasing in $M^I_s/M_s$. Given that $M^I_s/M_s$ is bounded between $[0,1]$, the policy function $M^I_s/M_s^i(M^I_s/M_s)$ must either cross the 45-degree line in the $(M^I_s/M_s^i, M^I_s/M_s)$ space, such that the BGP exists in the interior, or at $M^I_s/M_s = 1$ which is a corner solution. Given this result, and that consumption $c_n(X)$ is increasing in $M_s/M_n$ due to the restriction on $\psi_n$, the North’s problem also has the same BGP solution, either at the same interior point or when $M^I_s/M_s = 1$.

Proof of Proposition 3: Assume that the economy is on the BGP that is in the interior. Based on the value function $J_i(X)$ after solving for (45) and (47), with $\tilde{c}_i(X, \zeta_i(X))$ denotes the equilibrium consumption and $\zeta_i(\cdot)$ is the policy function, one can derive an Euler equation with the use of envelope theorems in Milgrom and Segal (2002), that $J_i(X)$ is differentiable almost everywhere except at the points where there are changes in pricing conditions listed in Table 1. By taking derivative of the value function with respect to $M_s/M_n$ for the North and $M^I_s/M_s$ for the South yields (see Appendix A.2 for derivation):

$$u'(\tilde{c}_i(X, \zeta_i(X))) = \frac{\beta u'(\tilde{c}_i(X', \zeta_i(X')))}{(1 + g_i(X))^{\theta-1}} \cdot \frac{(1 - \alpha^2)y_i'(X') - e'_i(X')}{r^*_i(X,X')}.$$  \hfill (A.1)

Focus on the South’s problem. Perturb $M^I_s/M_s^i < M^I_s/M^*_s$ in the next period. Given that consumption $c_s(X)$ is increasing in $M^I_s/M_s$, budget constraint (48) implies that the last term on the left-hand-side of (A.1) is positive and bigger at $M^I_s/M_s^i$ than at $M^I_s/M^*_s$. Consumption next period also drops as $\tilde{c}_s(M^I_s/M^i_s, \cdot) < \tilde{c}_s(M^I_s/M^*_s, \cdot)$. As such, $\beta u'(\tilde{c}_s(X', \zeta(X'))) > 0$. If today’s consumption is chosen to increase, so that $u'(\tilde{c}_s(X, \zeta(X)))$ decreases and investment drops so that so that $(1 + g_s(X))^{\theta-1}$ decreases, then the equality in (A.1) does not hold. Contrary, the equality holds if today’s consumption is chosen to decrease and investment to increase. $M^I_s/M_s$ moves back to $M^I_s/M^*_s$. Vice-versa is true if perturb $M^I_s/M^i_s > M^I_s/M^*_s$ in the next period.
As for the North’s problem, given that consumption $c_n(X)$ is increasing in $M_s/M_n$, one can use the same approach to show that perturbing $M_s/M_n < M_s/M_n^*$ reduces consumption $c_n$ and increases investment today so that $M_s/M_n$ moves back to $M_s/M_n^*$. Vice-versa is true by perturbing $M_s/M_n > M_s/M_n^*$.

Proof of Proposition 4: A strengthening in IPR increases $\eta_{imit}$, which reduces the growth rate $g_s(X)$ for the same R&D expenditure. The Southern household needs to choose the share of innovators ($M_I^s/M_s$) and hence R&D expenditure (see (38) and (48)) to avoid or minimize the decline in the overall growth path of $g_s(X)$ while smoothing consumption. $\psi_s^f \geq 0$ and $\psi_s^{imit} \geq 0$ guarantee an increase in expected profit $\hat{\pi}_{it+1}$ in (31). In addition, the capital gains from innovation and imitation are non-negative, which provide incentives for R&D investment along the transition path.\[23]\ But as seen in (32), in order to guarantee that the expected rate of return on investment ($\mu_s$), and hence $g_s(X)$, to increase after a strengthening in IPR, $\hat{\pi}_{it+1}$ must increase faster than capital gains in all future periods, such that $\hat{\pi}_i/V_i$ increases in the new BGP.\[24]\ The increase in the relative cost of production $\gamma$ between North and South in (11) cannot be too rapid, which would otherwise reduce the proportion of firms that can successfully survive the second stage problem to a level that would reduce the growth of the expected profits below capital gains. One can find $\bar{\sigma}$ that bounds $\sigma$ to be small enough so that $\gamma$ does not increase at an excessively fast pace. In sum, if the negative relative cost effect on firm distribution is dominated by the increase in expected profits associated with increasing innovation and the capital gains, then growth ($g_s(X)$) would increase as IPR strengthens $\forall X$.

Proof of Proposition 5: Assume that the the South has an initial IPR parameter of $b_{s0}$. To begin, assume that there are only two periods, transitional and BGP, and make use of equation (A.1) and apply it to the two periods. First, focus on the South’s problem. As IPR increases from $b_{s0}$, the cost of imitation $\eta_{imit}^i$ from (30) increases. Following Proposition 4, $(1 + g_s(X))^{\theta-1}$ in equation (A.1) increases. Given that $\bar{c}_i$ is an increasing function, $\beta u'(\bar{c}_s(X',\zeta_s(X'))) \text{ decreases}$, so that the first term on the right-hand-side decreases, and the numerator in the last term on the right-hand-side increases due to the budget constraint (48). Whether $u'(\bar{c}_s(X,\zeta_s(X)))$ on the left-hand-side increases, implying an decrease in transitional consumption, depends on whether investment in R&D grows excessively in the BGP from an optimal policy standpoint. Larger R&D investment at each $X$ means higher growth rates, and given that the R&D cost functions $\eta$ in

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\[23\] The capital gain channel only appears in transition and does not affect the BGP growth rate because capital gain is zero along the BGP.

\[24\] From (32), for $\mu_i$ to increase after a strengthening in IPR, the increase in $\bar{\pi}_i$ cannot be slower than the increase in $V_i$ along the BGP, and the increase in $\bar{\pi}_{it+1}/V_{it+1}$ cannot be slower than the increase in $\Delta V_{it+1}/V_{it+1}$ along the transition.
(29) to (30) are increasing and concave, the upward shift in the aggregate R&D schedule as IPR strengthens means that $r_s'(X, X')$ decreases. I refer to this as the “resource allocation effect”.

By conducting a policy experiment by continuously increasing IPR, one can find that after a certain IPR threshold, the smaller increase or even decrease in $r_s'(X, X')$ — the resource allocation effect — can increase the last term in the right-hand-side of (A.1) enough to more than offsetting the increase in $(1 + g_s(X))^{\theta - 1}$ and the decrease in $\beta u'(\bar{c}_s(X', \zeta_s(X')))$, in which I refer the two effects together as the “variety expansion effect”. As such, the equality brings an increase in $u'(\bar{c}_s(X, \zeta_s(X)))$ and hence a decrease in transitional consumption. This means that before certain IPR threshold, increasing $b_s$ above $b_{s0}$ causes transitional consumption to increase, but decreases it after the IPR threshold. In other words, transitional consumption as a function of $b_s$ is non-monotonic. We extend this logic from the two-period setting to infinite horizon by breaking the transitional period into the number of periods before the BGP is attained given the initial state $X_0$. Using equation (A.1), one can find a $b_s^\ast$ such that for $b_s \in (b_{s0}, b_s^\ast)$, the sum of lifetime discounted utility in the South increases, and for $b_s > b_s^\ast$, the sum of lifetime discounted utility decreases due to a significant drop in transitional consumption.

Next, for the North’s problem, an increase in Southern IPR induces a fall in the probability of imitation $\phi$ which in turn results in less imports from Southern imitators. But future $y_n$ could increase due to the rise in future innovation in the South and also increased incentive for Northern innovation due to a lower probability of imitation. Following the similar logic as above, using equation (A.1) one can find a $b_{s}^{**}$ such that for $b_s \in (b_{s0}, b_s^{**})$, the sum of lifetime discounted utility in the North increases, and for $b_s > b_s^{**}$, the sum of lifetime discounted utility decreases due to a significant drop in transitional consumption.

However, $b_s^\ast$ can be different from $b_{s}^{**}$. We can find $b_{s}^{***}$ to maximize the joint welfare of the two economies by assigning weights on the two value functions following the same approach above to find $b_s^\ast$ and $b_{s}^{**}$.

**Proof of Proposition 6:** Given that consumption $c_i$ increases with $y_i$, a reduction in $\tau$ increases the proportion of exporters, and hence imports, in both countries, and the average productivity $\bar{a}_i^{1-\epsilon}$ also increases as countries are importing inputs with an average productivity higher than the domestic inputs. Given the assumptions on the $\psi$ parameters in $\eta$ functions, R&D spending do not increase excessively to reduce consumption upon policy changes, so that consumption is increasing in $M_s/M_n$ in the North and $M_s^I/M_s$ in the South.
A.2 Deriving the Euler Equation

The value function $J_i(X)$ has kinks or jumps where there are changes in pricing conditions listed in Table 1. For example, when the pricing condition changes from (ii) to (iii), all Southern imitators exit the North and Northern innovators whose products have been imitated will re-enter the North market. Also, when the pricing condition changes and from (vi) to (vii), all Southern imitators exit the South’s market and Northern exporters whose products have been imitated will re-enter the South market. Based on envelope theorems described in Milgrom and Segal (2002), an Euler equation can be derived for a value function that is differentiable almost everywhere.

I drop the $i$ subscript in below, and add subscripts to denote the derivative with respect to $X$ and $X'$. Denote $\bar{c}(X, \zeta(X))$ the equilibrium consumption and $\zeta(\cdot)$ the policy function. Based on the value function $J(X)$ after solving for (45) and (47), taking the first order condition with respect to $X'$ yields:

$$J_{X'}(X) = u_{X'}(\bar{c}(X, \zeta(X)))(\bar{c}(X, \zeta(X)) + \bar{c}(X, \zeta(X))\zeta_X(X)) + \beta(1 + g(X))^{1-\theta}J_{X'}(\zeta(X)) = 0.$$ 

Differentiate $J(X)$ with respect to $X$ yields the envelope condition:

$$J_X(X) = u_X(\bar{c}(X, \zeta(X)))(\bar{c}_X(X, \zeta(X)) + \bar{c}(X, \zeta(X))\zeta_X(X)) + \beta(1 - \theta)(1 + g(X))^{-\theta}g_X(X)J(\zeta(X)) + \beta(1 + g(X))^{1-\theta}J_X(\zeta(X))\zeta_X(X),$$

where $\zeta_X(X) = 0$ and $g_X(X) = 0$ at optimal. Note that $\zeta_{X'}(X) = -r_{X'}(X, X')$ in the first order condition and $\bar{c}_X(X) = (1 - \alpha^2)y_X(X) - e_X(X)$ in the envelope condition. Combining the first order condition and the envelope condition at $X'$ yields:

$$u_{X'}(\bar{c}(X, \zeta(X))) = \beta(1 + g(X))^{1-\theta}u_{X'}(\bar{c}(X', \zeta(X')))(1 - \alpha^2)y_{X'}(X') - e_{X'}(X') \frac{r_{X'}(X, X')}{r_{X'}(X', X')}.$$
A.3 Numerical Solution Method

The optimization problems for the North and the South are given by the Bellman equations (45) and (47). The North’s problem can be solved without iterating the value function. Even though the North is choosing \( M_s/M_n \), \( M_s' \) is indeed given by the South’s problem, so the North only needs to choose the rate of innovation to get \( M_n' \) given \( M_n \). Given the assumption that the US economy is around the BGP, the solution can be approximated by the Euler equation at a given state \( X \).

I use value function iteration to solve for the South’s problem, taking Northern growth as given by the North’s problem. Instead of iterating through the entire \{\( M_s/M_n \), \( M_s'/M_n' \)\} space, I only need to solve for the policy function \( M_s'/M_s \) for a subset of \( M_s/M_n \) given by the sequence of \{\( M_s/M_n \)\} \( t=0,T \) where \( T \) is the time when the BGP is attained. In particular, after solving for the policy functions \( M_s'/M_s \) and \( M_s/M_n' \) at time \( t \), I treat these two as the state at time \( t+1 \), and solve for the new \( M_s'/M_s \) and \( M_s/M_n' \) which will be treated as the state at \( t+2 \), and so on.

The specific procedure is given as follows:

1. Define the grid of \( M_s'/M_s \) between 0.2 and 0.99.
2. Set up an outer loop for \( t = \{0,\ldots,\bar{T}\} \). Guess a big enough \( \bar{T} \) to ensure BGP is attained before \( \bar{T} \).
3. Given the parameters and state \( M_s/M_n \) at time \( t \), compute \( y_i \) and \( e_i \) for all \( M_s'/M_s \).
4. Set up an inner loop for the convergence of North’s problem. Guess an initial Northern firm’s capital gain.
5. Compute \( \mu_n' \) from (32) using the guessed capital gain for \( \Delta V_n/V_n \) instead of computing it from \( \eta_n'/\eta_n \). Given the almost-flat \( \eta_n \) function in (28) with \( \psi_n \) being close to 0, the use of guessed capital gain and loop over its convergence speeds up the numerical solution without the need to guess and loop over \( M_s/M_n' \). I use the consumption Euler equation to compute consumption growth (\( g_c^c \)) to approximate the rate of innovation (\( g_n \)), which is given by:

\[
g_n^c = \frac{\Delta C_n'}{C_n} = (\beta(1+\mu_n'))^\frac{1}{\theta} - 1
\]

6. Solve for South’s problem (47) using value function iteration. Compute \( g_{st}^I, g_{st}^J, \) and \( g_{st} \) using (49)-(51). Compute \( M_s/M_n' \) by \( M_s/M_n \times (1+g_s)/(1+g_n) \).

7. Approximate Northern firm’s capital gain by taking the log difference between \( \eta_n(M_s/M_n') \) and \( \eta_n(M_s/M_n) \) from (28). If the difference between new and old capital gains is smaller than the convergence criterion, then update \( M_s'/M_s = M_s'/M_s' \) and \( M_s/M_n = M_s/M_n' \) as the new state at time \( t+1 \) and repeat steps (3) to (7). Otherwise, update the guess of capital gain \( \Delta V_n/V_n \) with the new capital gain computed, then repeat steps (5) to (7).
References


